Wave Reflection of irregular waves from Multi-Layer Berm Breakwaters

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ABSTRACT

One of the emerging issues from an efficacious design of berm breakwaters is the estimation of wave reflection. In the present study, the wave reflection of a multi-layer berm breakwater (MLBB) has been studied based on model experimentation. To gain this goal, two-dimensional model tests have been carried out in a wave flume at Tarbiat Modares University. Irregular waves were generated using the JONSWAP spectrum. The effect of various parameters like wave height, wave period, water depth, and berm elevation from still water level is investigated on the wave reflection. The achieved outcomes of this study proposes a new formula estimating MLBBs wave reflection. Finally, the performance of the derived formula against the existing formulae proposed by other researchers is checked thoroughly. The results of the statistical evaluation indices revealed that the predicted wave reflection using the new formula is more accurate than the existing ones. Thus, it would be obvious that the present formula can provide a well-founded estimation of wave reflection on the MLBBs. Moreover, the new formula and those estimated by existing ones are validated by employing the data set exclusively used in drawing the comparison. This fair validation illustrates that the current formula is more accurate than the existing formulae.

1. Introduction

In the early 1980s, berm breakwaters had been evolved by accepting the reshaping concept in rubble mound breakwaters in the course of wave action on the front slope. Berm breakwaters are a particular type of rubble mound breakwaters built with a porous berm above the still water level at the seaward side [1]. This type of structure can be divided into two disparate categories encompassing homogeneous berm breakwaters (HBBs) and multi-layer berm breakwaters (MLBBs). The HBB’s armor layer comprises only one stone grading, whereas the MLBB’s armor layer consists of several stone classes with unique stone gradation [2]. Since MLBBs have been typically constructed in Iceland – over 20 cases so far - they are also celebrated as Icelandic berm breakwaters [3]. Due to the wave attack, these structures perform more or less as statically stable structures, depending on the stability number. Moreover, the armor layer stones are sorted into diverse classes in accordance with their sizes [4]. The overwhelming advantage of using various classes in the structure could be the extensive utilization of dedicated quarries [5]. The largest rock size is categorized as ‘class I’, located in front of the berm, and even sometimes at the upper slope of the berm, which plays a quite prominent role in the stability of the structure. On the contrary, the smaller stone classes are placed in the lower parts of the slope and inner layers, where the influence of the wave is not significant. Typically, these classes are from the same dimensions of rocks used in the armor layer of HBBs [3]. Figure 1 demonstrates the different parameters schematically in an MLBB. Overall, one can express the advantages of MLBBs, including the utilization of dedicated quarry optimally, narrow stone gradation, and providing a high level of porosity, dissipating the wave energy greatly, lowering the stone gradation, and providing higher interlocking. Hydraulic responses such as wave reflection, wave overtopping, run-up and run-down, and wave transmission are assumed to be determining factors in the design process of disparate coastal structures. Figure 2 gives an excellent display of these parameters...
[6]. Providing further details, it is self-evident that the incident wave energy would definitely end up with the reflection, dissipation and/or even transmission into the bay. The reflected wave from natural beaches or even human-made structures would considerably affect the sediment movements and hydrodynamics created at the toe of the structure [7]. Concerning this fact, a meticulous study into the nature of wave reflection in marine structures is crucial to be conducted. In the case of high reflection, the interaction of incident and reflected waves could potentially bring an extremely chaotic area into existence all along with very steep, standing, and breaking waves. Accordingly, on the one hand, it would be difficult to navigate in front of the bay entrance for smaller vessels in the presence of high reflected waves. On the other hand, these waves would increase the scouring level at the toe of the structure, the erosion at nearby beaches, and eventually behave towards the destabilization of the structure [8, 9]. On the whole, the prediction of wave reflection from coastal structures such as the rubble mound breakwaters is of utmost importance owing to its adverse effects [10].

The wave reflection is commonly expressed by the reflection coefficient ($C_r$). This coefficient is defined as the ratio of the reflected wave height to the incident wave height or of the root square ratio of the reflected wave energy to the incident wave energy as follows:

$$C_r = \frac{H_r}{H_i} = \sqrt{\frac{E_r}{E_i}} \tag{1}$$

where $H_i$ is the incident wave height, $H_r$ is the reflected wave height, $E_i$ is the incident wave energy, and $E_r$ is the reflected wave energy.

For the design of rubble mounds, the reflection coefficient is one of the most significant hydraulic replies of a breakwater. Over the past 60 years, a diversity of experimental formulae predicting $C_r$, primarily inspired by Miche’s approach [11] and expanded by Battjes [12], have been proposed. This approach presumes that the reflection process is governed by the wave breaking. First, investigations on the wave reflection date back to (1974) by Battjes, who theoretically derived a formula for the reflection coefficient [12]. He assumed $C_r$ as a function of Iribarren number ($\xi = \frac{\tan \alpha}{\sqrt{H_i/L_m}}$), and introduced Eq. (2) for smooth-impermeable slopes.

$$C_r = 0.1 \xi^2 \tag{2}$$

where, $\xi$ is the Iribarren number, $\alpha$ is the front slope angle, $L_m = gT_i^2/2\pi$ is the deep-water wave-length, $T_i$ is the mean wave period, and $g$ is the gravitational acceleration.

A large number of researchers have sought to determine the wave reflection on the rubble mound breakwaters and proposed or even elaborated some formulae to estimate the reflection coefficient, e.g., Losada and Gimenez-Curto [13], Seelig and Ahrens [14], Postma [15], Hughes and Fowler [16], Chegini et al. [17], Davidson et al. [8], Muttray et al. [18], Zanuttigh and Van der Meer [19, 20], Calabrese et al. [21], Zanuttigh and Lykke Andersen [10], Park et al. [22], Van der Meer and Sigurdarson [23], Mahmoudi et al. [24] and Buccino et al. [25].

Postma [15] carried out 300 tests on rock slopes to investigate the effect of wave period, wave height, front slope, water depth, permeability, stone gradation, and spectral shape on wave reflection. The results obtained from his study disclose the great dependency of $C_r$ upon wave period, front slope, and also the permeability, but marginal dependency on wave height and inconsiderable correlations with spectral form and water depth. More to his investigation, he proposed an empirical formula corresponding to

![Figure 1. Illustration of various parameters in an MLBB](image-url)
Battjes’ formula [12] in which:

\[ C_r = 0.14 \xi^{0.73} \]  

(3)

Commenting on the Iribarren number, Van der Meer [6] argued that it could not explain the effects of both wave steepness and front slope and also, as he mentioned, by means of the multiple regression analysis, a better fit to Postma’s data [15] is achievable. Overall, an empirical formula as the following form is derived:

\[ C_r = 0.081P^{-0.14} \cot \alpha^{-0.78} S_{op}^{-0.44} \]  

(4)

where, \( S_{op} = \frac{H_r}{gT_p^2/2\pi} \) is the wave steepness, \( H_r \) is the incident significant wave height, \( P \) is the permeability, and \( T_p \) is the peak spectral period. Van der Meer [26] has put forward the structure’s permeability, which is made up of the armor size, filter, and core layers from 0.1 for a relatively impervious core (sand or clay) to 0.6 in homogenous structures. Hughes and Fowler [16] presented a formula to estimate the wave reflection coefficient using the novelty of the Iribarren number concept as follows:

\[ C_r = \frac{1}{1 + 7.1 \xi^{0.8}} \left( \frac{d}{\sqrt{\frac{T^2 \tan \alpha}} \right) \]  

(5)

in which \( d \) is the water depth, and \( T \) is the wave period in regular waves. Buccino et al. [25] developed Hughes and Fowler [16] formula to predict a reflection coefficient formula for sloped coastal structures by taking the advantage of wave momentum flux approach, which was formerly introduced by Melby and Hughes [27]. Chegini et al. [17] studied wave reflection from rubble mound breakwater. The model tests were carried out on two different sections of breakwaters with armor layers of tetrapod units and antifer blocks. They stated that the coefficient of wave reflection increases with increasing the surf-similarity parameter. Lykke Andersen’s [3] study on berm breakwaters reveals that, as the slope and consequently the surf similarity parameter (\( \xi \)) differ, this would make it difficult to find a single value of (\( \xi \)) representing the breaking on the structure and also the phase lag between reflections from different parts of the structure. Van der Meer and Sigurdarsdson’s [23] study was conducted on the wave reflection with data being gathered and analyzed from other researchers’ investigations. Finally, they proposed the following formulae to estimate the wave reflection coefficient regarding Lykke Andersen’s [3] assumptions:

\[ C_r = 1.3 - 1.7 S_{op}^{0.15} \]  

(hardly and partly reshaping berm breakwaters)  

(6)

\[ C_r = 1.8 - 2.65 S_{op}^{0.15} \]  

(fully reshaping berm breakwaters)  

(7)

Owing to the unfavorable effects of wave reflection on coastal structures, their designing process lacks a requisite design criterion. A glance through previous studies indicates that there are some empirical predictive formulae merely encompassing the parameters by which the structure’s wave reflection was affected. Notwithstanding the major focus of the earlier studies on the wave reflection in conventional rubble mound breakwaters, it has not been given any undivided attention. Moreover, considering the structures in the company of a berm – with higher capability in energy dissipation compared to the conventional type – like MLBBs, the wave reflection value is smaller than the conventional rubble mound breakwaters. Since some berm parameters such as berm elevation from SWL were not taken into account, employing the conventional rubble mound breakwaters formula for the berm breakwaters would lead to considerable uncertainties. Moreover, since MLBBs generally perform as hardly/partly reshaping breakwaters, the wave reflection values are far away from fully reshaping berm breakwaters. It is worth mentioning that due to poor stones extracted from quarries in the south of Iran, conducting a comprehensive study on MLBBs (exploiting the quarry stones optimally) seems to be essential for deeper understanding and future practical cases. In the present research, a systematic experimental study was undertaken to examine the effect of various parameters on MLBBs wave reflection. To fulfill this task, a 2D experimental modeling in a wave-flume is considered to investigate the effects of wave height, wave period, water depth, and berm elevation, and finally, a proper formula is presented to estimate the wave reflection. The following structure of the study takes the form of six sections, including the experimental set-up, the range of dimensional and
dimensionless parameters, non-dimensional analyses, the effects of various parameters on wave reflection, the procedure of driving the proposed formula for the wave reflection coefficient, and in the final section the present formula is evaluated with the other existing ones.

2. Model test set-up
The present study was conducted in a wave flume in the hydraulic laboratory of the Faculty of Civil Engineering at Tarbiat Modares University equipped with a piston-type wave generator producing irregular waves (Figure 3). All experiments were conducted with irregular waves using a JONSWAP spectrum with a peak enhancement factor $\gamma=3.3$. To ultimately gain the definite wave combination, several tests have been repeated over and over again. Regarding the scale modeling, two highly permeable and active rock absorbers were set-up at both ends of the wave flume behind the paddle and also the opposite side to absorb the wave energy and consequently minimize the wave reflection impact on the newly waves generated by the paddle. The wave flume is $16 \times 1 \times 1 \text{m}$ (length x width x depth) with fitted glass panels throughout its length, providing convenient observations and filming. The MLBB experimental model consists of four stone classes inspired by the Sirevåg breakwater in Norway. Figure 4 shows the typical cross-section of the breakwater placed at the end of the flume.

![Figure 3. Longitudinal section of the model set-up](image)

![Figure 4. Typical initial cross section of the MLBB](image)

The literature review on reflection analysis reveals that in order to separate incident and reflected wave height, the known methods such as Goda and Suzuki [28] and Mansard and Funke [29] methods are exploited. Goda and Suzuki [28] method employs two wave probes at fixed positions to measure wave heights from both probes and the phase shifts between the two probes. This method fails when the spacing between the two wave probes is equal to an irregular number of half wave-length. In order to reduce the probes spacing problem and the sensitivity to noise, Mansard and Funke [29] proposed a method involving three gauges, which is based on a least-squares technique. In fact, the least-squares method overcomes the limitations of the two-point method, which was initially proposed by Mansard and Funke [29] for irregular waves. In this method, wave heights are measured from three probes aligned parallel to the wave propagation direction, and two groups of phase shifts among these gauges with certain distances apart are also measured. The main references showed that two-point method has limitations, such as limited frequency range, critical gauge positions causing singularity, and sensitivity errors. A least square method (e.g., Mansard and Funke’s [29] method) minimizes the squares of errors between measured and theoretical signals. It is less sensitive to these phenomena, and practically there is no limitation in its application range of frequency (or wave-length). Thus, it seems that Mansard and Funke’s [29] method indicates greater accuracy and range encompasses your mentioned restriction, and thereby it has been widely hired in many studies pertinent to irregular waves. In the present research, water level fluctuations were recorded with three wave gauges based on Mansard and Funke’s [29] pattern by which the incident waves and reflective waves are commonly separated. Wave gauges are a capacitive type with the frequency of 10 Hz and the precision of 1 mm, recording the water level fluctuations at every 100 milliseconds. As recommended by Mansard and Funke [29] the distance between wave gauges must satisfy the following limits:

$$X_{12} = L/10, \quad L/6 \leq X_{13} \leq L/3, \quad X_{13} \neq L/5, 3L/10 \tag{8}$$

where $X_{12}$ is the distance between the first wave gauge (wavemaker side) and the middle wave gauge, $X_{13}$ is the distance between the first wave gauge and the third wave gauge (breakwater side). A code was written in

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MATLAB, separating both reflective and incident waves according to Mansard and Funke’s method [29] soon after each test came to an end. Moreover, the main assumption underlying the analysis of reflection in an irregular sea state is that the irregular waves can be described as a linear superposition of an infinite number of discrete components each with their own frequency, amplitude, and phase. The given assumption has been widely accepted as a common fact in studying irregular waves. Due to this fact, these assumptions and Mansard and Funke’s [29] method have been used in the present research.

In order to ensure the accuracy of the constructed structure, it was properly recorded using a vertical point gauge in five distinct sections at distances of 0.02 m. The middle section is located in the middle of the flume, and the side sections are located at a distance of 0.15 m from each other. Moreover, throughout choosing the sections, in order to control the effect of walls on test results, the allowable distance from the sections to flume walls has been taken into account. Finally, the average of all five sections is considered as the initial and reshaped profile for further calculations. Table 1 lists the material properties, including the density of the armor, stone nominal diameter, and stone gradation in different layers for all tests. Figure 5 displays a view of the tested MLBB.

### 3. Range of dimensional and dimensionless parameters

The present experiments are designed to investigate a variety of parameters such as wave height, wave period, water depth, and berm elevation. The range of dimensional and non-dimensional parameters are given in tables 2 and 3. In this research, a wide range of variations for the dimensionless parameters such as Reynolds number, wave steepness, and surf similarity parameter are considered. For all the conducted tests, as the stability number varied within the range of 1.3-2.3, the structure performed as a partly/hardly reshaping berm breakwater.

#### Figure 5. A view of the MLBB model

![A view of the MLBB model](image)

### Table 2. Range of dimensional parameters in present experimental

<table>
<thead>
<tr>
<th>Variable</th>
<th>Range of parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Incident significant wave height ($H_s$)</td>
<td>0.059 to 0.103 (m)</td>
</tr>
<tr>
<td>Peak wave period ($T_s$)</td>
<td>1 to 1.54 (s)</td>
</tr>
<tr>
<td>Water depth (d)</td>
<td>0.24 to 0.28 (m)</td>
</tr>
<tr>
<td>Berm elevation ($h_{br}$)</td>
<td>0.03 to 0.07 (m)</td>
</tr>
<tr>
<td>Berm width (B)</td>
<td>0.2 (m)</td>
</tr>
</tbody>
</table>

### Table 3. Range of non-dimensional parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Range of parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>3000</td>
</tr>
<tr>
<td>$S_{op}$</td>
<td>0.016 to 0.065</td>
</tr>
<tr>
<td>$cota$</td>
<td>1.5</td>
</tr>
<tr>
<td>$L_{ref}$</td>
<td>5.57 to 15.4</td>
</tr>
<tr>
<td>$h_{br}/H_s$</td>
<td>0.1 to 1.18</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>1.97 to 6.33</td>
</tr>
<tr>
<td>$R_s$</td>
<td>1.82x10^4 to 3.0x10^8</td>
</tr>
</tbody>
</table>

Dai and Kamel [30], Thomsen et al. [31], Broderick and Ahrens [32], Jensen and Klinting [33], Burchart and Frigaard [34], Van der Meer [26], and Wolters et al. [35] conducted some experimental research on the scale effects, in which the Reynolds number was employed as the following equation to evaluate viscous scale effect in the experimental models of rubble mound breakwaters.

$$R_s = \frac{\sqrt{gH_s D_{ns50}}}{\nu}$$  \hspace{1cm} (9)

where $\sqrt{gH_s}$ is the characteristic velocity, $D_{ns50}$ is the characteristic length, and $\nu$ is the kinematic viscosity. The viscous scale effect could be neglected since the minimum Reynolds number for current tests is 2x10^4, which is greater than (1-4)x10^4 recommended by Van der Meer [26].

### 4. Non-dimensional analysis

Generally, to take the preliminary steps in simulating modelling a physical phenomenon, the decisive factors must be truly recognized. Since the physical
quantities can be defined in terms of dimensions, it is perfectly efficient to utilize a method creating dimensionless parameters with physical principles. One of the initial agents in experimental studying is dimensional analysis. This method would decrease the complexity and number of effective experimental parameters in a physical phenomenon using a compounding technic and therefore propose the dimensionless relations and relevant experimental results. The Buckingham Pi theorem is used as one of the comprehensive theories in dimensional analysis, to determine the dimensionless parameters. Afterwards, by changing the variables during the tests, we can check out their effects on the wave reflection of MLBBs and eventually derive appropriate formulae. Variables describing the sea state, armor units, and the structure geometry are illustrated in Eq. (10).

\[ F(H_s, H_r, T_p, d, N, \rho_w, \mu, g, h_b, \alpha, B, R_s, G_s) = 0 \]  

where \( \rho_w \) is the density of water, \( \mu \) is the dynamic viscosity of the fluid. Supposing a non-over topped structure, the height of the crest above the mean sea level, \( R_s \), and the width of the crown of the breakwater, \( G_s \), are not important. Also, the front slope angle (\( \alpha \)), the berm width (\( B \)) and the number of waves (\( N \)) are all remained constant in all tests. Hence, Eq. (10) can be reduced as follows:

\[ F(H_s, H_r, T_p, d, \rho_w, \mu, g, h_b) = 0 \]  

Eight variables remained in Eq. (11). By using the Buckingham Pi theorem and compounding method, 5 (=8 – 3) independent dimensionless parameters are obtained as the following equation:

\[ X\left(\frac{H_s}{H_r}, \frac{\rho_w g H_s}{\mu}, \frac{S_o}{d}, \frac{L_o}{H_s}, \frac{h_b}{H_s}\right) = 0 \]  

The second parameter is the Reynolds number (\( R_e \)), which has negligible influence based on the aforementioned discussion, so it is eliminated from Eq. (12). Thus, Eq. (13) can be written as follow:

\[ C_e = \Psi\left(\frac{L_o}{d}, \frac{h_b}{H_s}\right) \]  

Eq. (13) conveys the fact describing the wave reflection from an MLBB for considered wave conditions and structure’s variables.

5. Results and Discussion

As stated earlier, one of the critical issues in studying the functional behaviors of the rubble mound structures is the wave reflection of the breakwaters. In the following section, at first, the effect of different parameters on the MLBB’s wave reflection is discussed in accordance with present experimental tests. Afterwards, by analyzing the test results, a new practical formula to estimate the wave reflection is derived for the MLBB. Note that the structure is considered as a non-over topped breakwater all through the tests.

5.1. Effect of wave height on the wave reflection

In the current study, the effect of wave height on wave reflection for four disparate wave heights (\( H_s = 0.059 \), 0.0745, 0.09, and 1.03 m) is investigated. As a point to note, the water depth, the berm elevation, and the wave number are constant values of 0.28 m, 0.05 m, and 3000 waves, respectively. Figure 6 indicates the influence of wave height on wave reflection in four disparate wave periods. Figure 6 pinpoints the marginal effect of wave height on wave reflection coefficient. The observed smooth decrease in the wave reflection coefficient could be attributed to an increase in some of the given data wave height. As a reason for the mentioned behavior, Postma [15] stated that energy dissipation along the slope is due to the existing drag forces. These forces are directly increased with the square of the local wave-particle velocities, which in turn increase as wave height increase. As another pertinent study to this case, Muttray et al. [18] expressed the effect of wave breaking and permeability, which are respectively defined by a decrease and an increase in wave reflection coefficient in the presence of raised wave height, are approximately balanced. Thus, the reflection coefficient is approximately independent of wave height.

5.2. Effect of wave period on the wave reflection

Throughout this section, the influence of the wave period on the wave reflection of the MLBB is investigated in four different wave periods (\( T_p = 1, 1.18, 1.36, \) and 1.54 s) with the same heights. The water depth (\( d = 0.28 \) m), the berm elevation (\( h_b = 0.05 \) m), and the wave number (\( N = 3000 \)) are constant during all wave periods tests. Figure 7 plots the wave reflection coefficient versus the wave period for several values of wave heights. Figure 7 highlights the supreme importance of the wave period, owing to a remarkable increase in wave reflection coefficient. This result has also been concluded by other researchers such as Scheffer and Kohlhass [36].

![Figure 6. Influence of wave height on wave reflection coefficient for disparate wave periods](image-url)
5.3. Effect of berm elevation on the wave reflection

A glance through the recent research on wave reflection discloses that nearly several studies were exclusively aimed at the conventional rubble mound structures, in which berm structural parameters such as the berm elevation were not considered. However, in this paper, 12 tests were carried out to investigate the effect of the berm elevation on the wave reflection of an MLBB constructed with three different berm elevations \((h_b=0.03, 0.05, \text{ and } 0.07 \text{ m})\) for different wave combinations. It is noteworthy to state that all through the tests probing into the effect of berm elevation on MLBBs’ wave reflection, other effective parameters such as water depth, the height of class I must be constant, while the berm elevation from the structure’s toe is changed from the lower levels by adding or removing stone classes II or III [37].

Figure 8 depicts the influence of berm elevation on the wave reflection coefficient for different wave combinations. As it is shown, increasing the berm elevation, the value of the wave reflection coefficient will rise. It can be concluded from observations that at the SWL close to the berm level, the waves in the process of run-up were distributed widely on the porous berm and even the upper slope of the berm resulting in higher wave energy dissipation. Consequently, by increasing dissipation, the wave reflection of the structure will be reduced. However, in the higher berm elevation from SWL, the berm performs inefficiently in wave energy dissipation. In the case in fact, the MLBB acts as a porous obstacle against the wave attack similar to conventional rubble mound breakwaters and leads to a higher wave reflection. Ehsani et al. [38] meticulously investigated the case of how the berm elevation can affect the wave energy dissipation in MLBBs.

5.4. Effect of water depth on the wave reflection

Reviewing the literature concerns the effect of water depth on the wave reflection of the rubble mound breakwaters depicts that this parameter affects the structure’s wave reflection. In order to obtain reasonable and accurate results in an experimental study of water depth in berm breakwaters, two particular points must be noted:
1- In the case of maximum water depth at the toe of the structure, the breakwater should behave as a non-overtopped structure.
2- The height of stone class I and berm elevation from still water level must be kept constant as the water depth changes.

Over this investigation, the effect of water depth on the wave reflection of the MLBB has been examined at three different water depths \((d=0.24, 0.26 \text{ and } 0.28 \text{ m})\). Pre-tests were also conducted to control the breakwater performing as a non-overtopping structure. It is worth to note that in order to examine the effect of the water depth, the berm elevation from SWL must be maintained constant (i.e. \(h_b=0.05 \text{ m}\)). It is apparent from Figure 9 -illustrating the influence of the water depth on the wave reflection coefficient-that an increase in water depth corresponds to the decrease in the wave reflection coefficient.

![Figure 7. Influence of wave period on wave reflection coefficient for different wave heights](image1)

![Figure 8. Influence of berm elevation on wave reflection coefficient for different wave combinations](image2)

![Figure 9. Influence of water depth on wave reflection coefficient for different wave combinations](image3)
6. The formula derivation methodology

So far, many reflection formulae have been proposed as a function of the Iribarren number. If the front slope is assumed as a constant parameter in the Iribarren number, the wave steepness can be defined as an effective parameter. Figure 10 displays the effect of the wave steepness against the wave reflection coefficient.

Figure 10 denotes a fairly high scattering in the data set, and the trend reveals that an increase in the wave steepness causes a decrease in the wave reflection coefficient. Based on the aforementioned discussion, an independent wave height approach should be utilized, in which the following dispersion equation is firstly taken into account:

\[ \sigma^2 = g k t g h (k d) \]  \hspace{1cm} (14)

where \( k \) is the wave number \( (K=2\pi/L) \), and \( \sigma \) is the angular frequency of the wave. The dispersion equation expresses that the wave number and the angular frequency of the wave are not independent. On the other hand, the dispersion equation justifies the effect of wave period by means of a parameter with length dimension such as \( L_o \) or \( T^2/d \). Since the dimension of \( L_o \) is length, it is highly appropriate to express the effect of wave period by means of a parameter with length dimension such as \( L_o \). Thus, in the present study, \( L_o/d \) is used as a non-dimensional parameter. As detailed in Figure 11 illustrating the wave reflection coefficient versus \( L_o/d \), \( C_r \) has a considerable reliance on \( L \). In order to scrutinize the effect of \( L_o/d \) on the wave reflection, several algebraic functions are employed, and finally, the power function is chosen as follows:

\[ C_r = \lambda (L_o/d)^6 \]  \hspace{1cm} (15)

To find the coefficients \( \lambda \) and \( \eta \) - denoting the shape and the curve trend- a nonlinear regression is used for each combination of \( h_{br}/H_s \). Due to the same trend variations of the wave reflection coefficient in discrete \( h_{br}/H_s \), this coefficient has small discrepancies, so an average value of \( \eta = 0.5 \) is used in the final equation. Note that the value of \( \lambda \) is a function of \( h_{br}/H_s \). Ultimately, a suitable pattern considering the effect of \( L_o/d \) on wave reflection coefficient will be used as follows:

\[ C_r = \lambda (L_o/d)^{0.6} \]

(16)

Trying to predict the value of the variable \( f(h_{br}/H_s) \), Eq. (16) is rewritten as the following equation:

\[ f(h_{br}/H_s) = \frac{C_r}{(L_o/d)^{0.6}} \]  \hspace{1cm} (17)

In order to determine the value of \( f(h_{br}/H_s) \), the right side of Eq. (17) against the corresponding \( h_{br}/H_s \) is plotted in Figure 12.

To gain a proper estimation function of the variables \( f(h_{br}/H_s) \), several algebraic functions are examined. Ultimately, the power function for the present experimental data is proposed as an appropriate model. By employing regression analysis, the new formula can be written to predict the wave reflection coefficient for MLBB as follows:

\[ C_r = 11.21(h_{br}/H_s)^{0.12}(L_o/d)^{0.6} \]  \hspace{1cm} (18)

Following the present experimental work limitations, the range of measured values are as Eq. (19), and therefore, the new formula is valid for this range.

\[
\begin{align*}
S_o &= 0.016 - 0.065 \\
\xi &= 1.97 - 6.33 \\
h_{br}/H_s &= 0.29 - 1.18 \\
L_o/d &= 5.57 - 15.4 \\
cot\alpha &= 1.5
\end{align*}
\]  \hspace{1cm} (19)
7. Validity assessment of the present formula with other researcher’s formula

Statistical validation indices are hired to validate the performance of the new formula and other researcher’s formulae. To fulfill this achievement, the present formula is compared to other formulae such as Postma [15] and Hughes and Fowler [16] based on the present experimental data. In order to evaluate the efficiency of various methods, different validation indices such as the square of the correlation factor \( R^2 \), the normalized root mean square error \( NRMSE \), and Bias are employed as follow:

\[
R^2 = 100 \left( \frac{N \sum XY - (\sum X)(\sum Y)}{\sqrt{[N \sum X^2 - (\sum X)^2][N \sum Y^2 - (\sum Y)^2]}} \right)^2 \\
NRMSE = \sqrt{\frac{\sum (Y - \bar{Y})^2}{\sum (Y - \bar{Y})^2}} \\
BIAS = \bar{X} - \bar{Y}
\]

where \( X \) is the calculated value, \( \bar{X} \) is the average value for the calculated data, \( Y \) is the observation data, \( \bar{Y} \) is the average value for the observation data, and \( N \) is the total data.

On the basis of present experimental data, table 4 tabulates the value of the evaluation indices and demonstrates that the present formula efficiently predicts the wave reflection coefficient rather than the other formulae. It is worth noting that the value of negative Bias in table 4 shows different methods underestimating the wave reflection coefficient.

Furthermore, it could be drawn that the other researcher’s formulae show a relatively high scattering, which could be attributed to disregarding some effective parameters such as the water depth at structure’s toe. In addition to the given regard, it is worth mentioning that the methods extended by Hughes and Fowler [16] were only unique to statically straight slopes without a berm. However, as we mentioned earlier, the berm elevation from SWL is known as a remarkably effective parameter on the wave reflection.

More details on validation are given in this section by using the rest of the present experimental data to provide an efficient comparison between the proposed equation and existing ones, such as Postma [15] and Hughes and Fowler [16]. It is remarkable to note that to make a fair comparison, the mentioned experimental data is not exploited in the new formula (Eq. (18)) derivation. Figure 14 depicts the comparison of measured and predicted wave reflection coefficient based on the aforementioned methods. It is easily observed that the proposed wave reflection formula matches perfectly with the current data and illustrates less error in contrast with other formulae results. Thus, considering some parameters such as berm elevation and water depth would obviously explain why there has been less error in the proposed formula, which performs independently from the wave height. It is necessary to point out that Postma [15] and Hughes and Fowler [16] methods investigated conventional rubble mound breakwaters. Thereby, the low accuracy of the other researcher’s formulae could be justified by some effective parameters that were disregarded in their formulae. To estimate the capability of the new formula in contrast with the existing ones, the verification indices for the given formulae are evaluated. Table 5 shows the validation indices for different formulae.

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<tbody>
<tr>
<td>( R^2 ) (%)</td>
<td>77.6</td>
<td>89.6</td>
<td>93.7</td>
</tr>
<tr>
<td>( NRMSE )</td>
<td>0.71</td>
<td>0.8</td>
<td>0.25</td>
</tr>
<tr>
<td>( Bias )</td>
<td>-4.26</td>
<td>-3.4</td>
<td>-0.1</td>
</tr>
</tbody>
</table>
Figure 13. Comparison of measured and calculated wave reflection coefficient for present data for various formulae

Figure 14. Comparison of measured and predicted wave reflection coefficient based on MLBB data for various formulae
Table 5. Validation indices for available formulae on wave reflection coefficient with MLBB experimental data

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<tbody>
<tr>
<td>$R^2$ (%)</td>
<td>31.4</td>
<td>33.2</td>
<td>75.9</td>
</tr>
<tr>
<td>NRMSE</td>
<td>1.42</td>
<td>0.99</td>
<td>0.67</td>
</tr>
<tr>
<td>Bias</td>
<td>-2.76</td>
<td>-2.42</td>
<td>0.04</td>
</tr>
</tbody>
</table>

In the following, the given formula estimating the wave reflection coefficient and other existing ones have been evaluated using Sveinbjörnsson’s [39] data set for a multi-layer structure. It is worthy to note that Sveinbjörnsson’s [39] study was conducted under 3000 number of waves and an armor slope of 1:1.5. Table 6 provides a comparison between the statistical evaluation of indexes in estimating wave reflection for Sveinbjörnsson’s [39] data set regarding our outcomes using Eq. (18) and other existing formulae. Moreover, as given in Fig. 15, the measured and calculated values of the wave reflection coefficient using the present formula have been compared to other researcher’s formula for Sveinbjörnsson’s [39] data set. One can conclude from the outcomes that our formula performs more appropriately than other proposed formula. It should be noted here that all other proposed formulae are unique to homogenous armored rubble mound breakwaters, while the present formula estimates the wave reflection for multi-layer armored breakwaters. As one can observe from the BIAS index, in comparison with average measured values, the calculated values using other researcher’s formula are overestimated. For instance, the BIAS parameter from Eq. (4) is equal to (6.16), and through the use of Eq. (5), it would be (4.69).

Table 6. Validation indices for available formulae on wave reflection coefficient with Sveinbjörnsson’s (39) data set for a multi-layer structure

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<tbody>
<tr>
<td>$R^2$ (%)</td>
<td>13.6</td>
<td>37</td>
<td>82.5</td>
</tr>
<tr>
<td>NRMSE</td>
<td>2.47</td>
<td>3.91</td>
<td>0.49</td>
</tr>
<tr>
<td>Bias</td>
<td>6.16</td>
<td>4.69</td>
<td>-0.61</td>
</tr>
</tbody>
</table>

Figure 15. Comparison of measured and predicted wave reflection coefficient based on Sveinbjörnsson’s (2008) data set for various formulae
8. Conclusions
This paper has sought to investigate the wave reflection of the MLBBs based on an experimental study and also examined the influence of different parameters including wave height, wave period, water depth, and berm elevation. Our work has led us to draw the given conclusion:
1- It has been noticed that there is no significant difference in the wave reflection coefficient with respect to different wave heights.
2- Variations in the wave period have a principal effect on the wave reflection coefficient so that as the wave period increases, the wave reflection coefficient would be considerably increased.
3- A novel formula (Eq. (18)) encompassing a wide range of non-dimensional parameters is derived to estimate the wave reflection coefficient of MLBBs.
4- The overwhelming advantage of the new equation compared to the other existing methods has been found to be due to its derivation on the basis of an independent wave height approach.

It should be highlighted that since the effect of water depth and berm elevation are both included in the present formula, it would perform more appropriately compared to other existing ones, at least for hardly/partly reshaping breakwaters. The conclusions from this equation indicate a more precise estimation rather than the other formula for predicting the wave reflection coefficient, at least for the MLBBs experimental data.

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9. References
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