INTERNATIONAL JOURNAL OF
COASTAL & OFFSHORE ENGINEERING
IJCOE Vol.5/No. 3/Summer 2021 (1-6)

Available online at: www. Ijcoe.org

Reliability analysis of Tension-Leg Platform Tendon with Respect to Fatigue Failure under Environmental Condition of Caspian Sea

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ARTICLE INFO

Article History:
Received: 03 Mar. 2020
Accepted: 29 June. 2021

Keywords:
Tendon
Tension-leg platform
Fatigue
FORM
Monte Carlo sampling

ABSTRACT

The primary objective of this paper is probabilistic quantification of the fatigue life of tension-leg platforms (TLP) using reliability methods. The need for such methods stems from the significant uncertainty in the loads exerted on offshore structures. The scope of this paper is limited to the study of fatigue in TLP tendons. For this purpose, nonlinear time-history of force response of the TLP tendon under random-wave load is computed via MOSES software and the damage due to fatigue is estimated in accordance with the Palmgren-Miners rule. Assuming a Rayleigh distribution for stress variation and eight different sea states, the ultimate fatigue damage is computed by accumulating the damage over all individual sea states. This cumulative damage enters the limit-state function that is based on the Palmgren-Miners rule. Prevailing sources of uncertainty in this problem are those in the estimation of fatigue stresses, fatigue strength, and the Palmgren-Miners rule. Finally, reliability analysis is carried out for four different service lives using the first- and second-order reliability methods (FORM and SORM) and Monte Carlo sampling. The results indicate that FORM computes the probability of failure sufficiently accurate. It is concluded that the probability of failure increases drastically with the service life. The importance vector from the sensitivity analysis in FORM reveals that the model error is the most influential source of uncertainty on the probability of failure.

1. Introduction

TLP's are floating structures anchored to the sea bottom through vertical tensioned tendons. The tendons hold the platform in place and they remain tensioned due to the excess of buoyancy caused by the floating hull. The tendon system is a critical component for the TLPs, since its failure may lead to the collapse of the whole structure involving human lives, economic losses and damages to the environment. Thus, the tendon system has to be designed to withstand the possible occurrence of different limit states like fatigue. Since offshore structures are subjected to random wave loading, which may contribute to fatigue failure, analyzing and making realistic predictions of failure probability is necessary.

Chatterjee et al. [1] developed a computational tool to handle hydrodynamic and structural aspects of TLP together. They also generated relevant information for a nonlinear static local stress analysis of TLP components from a dynamic hydro-structural global analysis. Lotsberg [2], Banon and Harding [3], Amanullah et al. [4], Khan and Siddiqui [5] studied reliability of TLP tendons under conventional environmental forces. The reliability assessment of TLP tendons under less probable small duration impulsive forces such as that arising due to collision of ships, icebergs, big marine or sea creatures, etc. Siddiqui and Ahmad [6] studied fatigue and fracture reliability of TLP tendons under random loading of sea waves. In this study, fatigue reliability of TLP tendons are evaluated using two common methods, i.e., first order reliability method (FORM) and Mont Carlo simulation. A more detailed application of reliability methods in fatigue assessment of existing offshore structures can be found in Gerhard [7]. Tabeshpour and Malayjerdi [8] stated the effect of pitch degree of freedom on the dynamic response of tendons that can affect the stress in tendons. Such fatigue can lead to tendon failure [9]. In this research, reliability analysis is performed using an approximate method, which models the structure directly as a system rather than modeling of the structural system as a system of components.

[DOI: 10.52547/ijcoe.5.3.1]

2. Limit state function

Fatigue failure has been defined through the limit state function g(z) which is negative or zero at failure. Z is vector of basic variables describing load, material properties, geometry variables, statistical estimates and model uncertainties. The probability of failure or probability of limit state violation is defined as

$$P_f = P[g(z) \ll 0] = \int_{g(z)<0} f_z(z)dz \qquad (1)$$

where $f_z(z)$ is the joint probability density function of vector z which is the product of individual probability density function of uncorrelated random variables. For fatigue of offshore platforms the major uncertainties involved are due to:

- Estimation of environmental parameters.
- Calculation of hydrodynamic and wind loads.
- Calculation of structural response.
- Calculation of local stresses (stress concentration factors) and stress intensity factors.

In the present study on fatigue reliability the following model has been employed for the formulation of limit state function.

2.1 Miner-Palmgren damage model

Equations in this approach, the fatigue strength is expressed in terms of S-N relation, which gives the number of stress cycles N with stress range S to cause failure. The S-N model generally used for high-cycle fatigue is given as

$$NS^m = A \qquad (2)$$

where S is the stress range; m, A are empirical constants; and N is the number of cycles causing failure. The TLP is subjected to environmental loadings, which are random in nature. Consequently, the tendon stresses are random in nature. The estimation of fatigue damage under stochastic loading is commonly done by the Miner-Palmgren model. In this model it is assumed that the damage on the structure, per load cycle, D_j is constant at a given stress range S_j and is equal to

$$D = \frac{1}{N(S_j)} \qquad (3)$$

Where $N(S_j)$ is the total number of cycles to failure at stress rangeS_j. The total damage accumulated in time T_s is thus given by

$$D = \sum_{j=1}^{N(T_s)} \frac{1}{N(S_j)} \qquad (4)$$

Where $N(T_s)$ is the total number of stress cycles in timeT_s. In this formulation, it is assumed that the accumulated damage D is independent of sequence in which stress cycles occur.

Using the S-N curve, the accumulated damage, D, is given as

$$D = \sum_{j=1}^{N(T_s)} \frac{S_j^m}{A} \qquad (5)$$

Since each stress range is a random variable $\sum_{j=1}^{NT_s} S_j^m$ is also a random variable. If $N(T_s)$ is sufficiently large, the uncertainty in the sum is very small and the sum can be replaced by its expected value. Therefore

$$E\left|\sum_{j=1}^{N(T_s)} \frac{S_j^m}{A}\right| = E[N(T_s)]E\left[S_j^m\right] \qquad (6)$$

For a narrow-band Gaussian process, stress ranges are Rayleigh distributed. The mean value of the stress range follows directly as

$$E\left[S_j^m\right] = \int_0^\infty (2x)^m \frac{x}{\sigma_x} \exp\left(-\frac{1}{2}(\frac{x}{\sigma_x})^2\right) dx = (2\sqrt{2})^m \sigma_x^m \Gamma(1+\frac{m}{2}) \qquad (7)$$

Hence, the accumulated damage is given as

$$D = \frac{1}{A} E[N(T_s)]E\left[S_j^m\right] \qquad (8)$$

If we consider the environmental condition being described as a set of stationary short-term sea states, the total damage can be obtained by summing the accumulated damage over all sea states. Thus, the total damage D yields:

$$D = \frac{T_s}{A}\Omega \qquad (9)$$

Where Ω is stress parameter given as

$$\Omega = (2\sqrt{2})^m \Gamma(1+\frac{m}{2}) \sum_{q=1}^{n} f_q \nu_{o_q} \sigma_q^m$$
$$\nu_{o_q} = \frac{1}{2\pi\sqrt{\frac{m_2}{m_0}}} \qquad (10)$$
$$\sigma_q = \sqrt{m_0}$$

Where ν_{o_q} is the zero mean crossing frequency of stress process in q[th] sea state, f_q is the fraction of time in q[th] sea state, σ_q is the standard deviation of stress process in q[th] sea state. m_q is the n[th] moment of stress spectrum.

$$m_q = \int_0^\infty \omega^n S(\omega)d\omega \qquad (11)$$

Failure occurs if $D > \Delta_F$ where Δ_F is the value of the Miner-Palmgern damage index at failure. Often Δ_F is taken as 1.

Letting $D = \Delta$, the time for fatigue failure T of a joint is obtained as

$$T = \frac{\Delta_F A}{\Omega} \qquad (12)$$

In order to take into account the uncertainties associated with the above expression, the factors involved in the expression shall be modeled as random variables. The time to failure T_i of joint I may be given as

$$T = \frac{\Delta_F A}{B_i^m \Omega} \qquad (13)$$

Where, Δ_F, A_i, B_i are random variable.

In the above Eq.(13), B_i describe the inaccuracies in estimating the fatigue stresses. The actual stress range is assumed equal to the product of B_i and the estimated stress range S. The uncertainties in fatigue strength, as evidenced by scatter in S-N data, are accounted by

[DOI: 10.52547/ijcoe.5.3.1]

considering A_i to be a random variable. The random variable Δ_F quantifies modeling error associated with Miner-Palmgren rule.

Uncertainty in fatigue stress estimates is assumed to stem from five sources. Attempts are made as follows to quantify the uncertainty contributed by each. A suggested model for B_i is [10]:

$$B = B_m B_s B_F B_N B_H \quad (14)$$

In which, each B_i is a random variable describing uncertainty as follows: B_M is fabrication and assembly operations; B_S is seastate description; B_F is wave load predictions; B_N is nominal member loads; B_H is the estimation of hot spot stress concentration factors. As the model error depends on various parameters it is found hard and time taking to determine it. Thus, according to the rules, a lognormal probability distribution function with specified mean and standard deviation is assumed for modeling error parameter [10]. The fatigue failure occurs when the random variable T_i is smaller than T_s where the T_s is the lifetime of the structure. Thus, the limit state function is

$$g(\underline{z}) = \frac{\Delta_F A}{B^m \Omega} - T_s \quad (15)$$

where

$$\underline{Z} = (\Delta_F, A, B) \quad (16)$$

The surface $g(\underline{z})$ is the limit state surface, and z is the vector of basic random variables in the problem.

The failure probability is computed using First order reliability method and Monte Carlo simulation technique. If

$$z_1 = \Delta_F \\ z_2 = A \\ z_3 = B \quad (17)$$

Then the limit state function is

$$G(\underline{Z}) = \frac{z_1 z_2}{z_3^m \Omega} - T_s \quad (18)$$

And the probability of failure P_f is

$$P_f = P(T_i \ll T_s) = P[G(z) \leq 0] \quad (19)$$

The reliability or safety index is thus obtained by

$$\beta = \phi^{-1}(P_f) \quad (20)$$

Where ϕ^{-1} , is the inverse of the standardized normal distribution function.

1.1. System reliability

TLP tendons are made of welded elements that form a series system. If any its joints fails, the system fails. If the failure of any joint of any member in a series system were mutually independent, the system probability of failure P_{fsys} can be formulated in terms of welded joints failure probabilities P_{fj} form basic probability considerations:

$$P_{fsys} = 1 - \prod_{j=1}^{n} (1 - P_{fj}) \quad (21)$$

And reliability index as is:

$$\beta_{sys} = -\emptyset^{-1}(P_{fsys}) \quad (22)$$

where n is the number of joints.

2.2 Wide band correction

Fatigue stresses are assumed narrow band random process. However, if they are wide band random process then the stress parameter Ω has to be modified accordingly through a correction factor. In the present study, Wirsching's wide band correction factor (λ) has been applied to modify the expression of Ω Therefore, the corrected expression for stress parameter (Ω) is:

$$\Omega = (2\sqrt{2})^m \Gamma(1 + \frac{m}{2}) \sum_{q=1}^{n} f_q \nu_{o_q} \sigma_q^m \lambda_q \quad (21)$$

Where λ is Wirsching's wide band correction factor for q^{th} sea state [10]. Estimates of λ_q is obtained by the following empirical expressions given by:

$$\lambda_q(\varepsilon_q, m) = a(m) + [1 - a(m)] \\ (1 - \varepsilon_q)^{b(m)} \\ a(m) = 0.926 - 0.033m \\ b(m) = 1.587 - 2.323 \quad (22)$$

And ε_q is the spectral width parameter for q^{th} sea state. For a typical ocean structure problem if ε_q >0.5, then λ_q ≅0.79 for m =4.38 and λ_q ≅0.86 for m =3.

3 Environment

In this study, environmental condition is limited to random waves only. According to data from Iranian Institute of Oceanography and Atmospheric Science, time series of significant wave-height for Caspian Sea is extracted. Thus, eight seastates are obtained based on the probability distribution function that is fitted over mentioned data [12] (Table 1).

$$\sum f(H_s)\Delta H_s \approx 1 \quad (23)$$

And zero up crossing period T_z is estimated as [13]

$$T = 3.21(H_s)^{0.5} \quad (24)$$

Table 1. Caspian Sea Seastates

Seastate	Significant Wave height	T_z	Occurrence Probability
1	1.18	3.46	0.314329
2	2.15	4.67	0.270879
3	3.65	6.09	0.251491
4	5.15	7.23	0.102182
5	6.65	8.22	0.033753
6	8.15	9.10	0.009534
7	9.69	9.93	0.002394
8	11.15	10.651	0.000499

4 Numerical Study

ISSC TLP as described in Table 2 was chosen for reliability study for eight simulated seastates [14]. Dynamic analysis was performed with MOSES software for long-crested random wave idealizing the platform hull as rigid body anchored to the seabed acting like springs. The analysis was carried out using three-dimensional diffraction theory. This non-linear dynamic analysis considers hydrodynamic loading due to random sea represented by JONSWAP spectrum.

[DOI: 10.52547/ijcoe.5.3.1]

The effects of wind and current was ignored in this study.

The stress time-series of the tendons were carried out for 500 seconds with the time step of 0.5 sec. The response was assumed to follow a zero mean Gaussian process. This assumption would be violated particularly if springing and ringing, a higher order wave effects, were considered in the analysis. These major non-linearity causing effects were not being considered in the present study. Figure 2 shows schematic arrangement of tendons under each column.

The stress time-series were statistically analyzed and statistical parameters of stress response were obtained (Table 3). Reliability analysis was carried out using Miner-Palmgren damage model. A brief description of statistics of the random variables are summarized in Table 4.

The probability of failure and reliability indices for four different service lives obtained for long crested random waves using Miner-Palmgren damage model are shown in Table 5. The reliability computations were performed utilizing RT software [16]. Three methods were applied for reliability analysis: First and second order reliability method and Monte Carlo simulation. Two of the most commonly used reliability methods are first and second order reliability. The basic idea of these methods is to ease the computational difficulties through simplifying the calculations and approximating the limit-state function. The name of First Order Reliability Method (FORM) comes from the fact that the limit state function is approximated by the first order Taylor expansion and the Second Order Reliability Method (SORM) uses the second order Taylor expansion. In Monte Carlo simulation, probability of failure is calculated by generating random numbers according to the probability distribution function of the random variables. Therefore, the outcome of the Monte Carlo simulation is considered as the accurate solution for the comparison. In RT, the maximum iterations and coefficient of variation were set ten million and 2% respectively for convergence criteria. Also for system reliability analysis, it was assumed that each tendon was made of 40 welded elements forming a series system. Stress time-history should be calculated for each joint to obtain system reliability but for simplicity, it was logical to assume that the statistic variables of stress at joints were equal.

3 Discussion of results

Figure 3 and 4 show that the FORM results have close proximity with Monte Carlo simulation results. Although SORM uses accurate approximations in comparison with FORM, its computations are often more complicated and time taking. Therefore, FORM is equally sufficient for such problems as Monte Carlo simulation method. Thus, it can be recommended for economical and efficient computation of reliability or probability of failure.

Service life directly affects the probability of failure or reliability of a system. It is seen that as the service life increases, the corresponding probability of failure increases drastically. This is an expected trend.

Table 2. Platform and tendon characteristics [14]

Characteristic	Value
Column spacing	86.25 $[m]$
Column diameter	8.44 $[m]$
Pontoon height	10.5 $[m]$
Pontoon width	7.5 $[m]$
Vertical center of gravity	38 $[m]$
draft	35 $[m]$
Mass of platform	$40.5*10^6$ $[kg]$
Depth	800 [m]
Number of tendons	12
Total pretension	$137.2*10^6$ [N]
Length of tendon	765 [m]
Tendon outer diameter	600 [mm]
Tendon wall thickness	88 [mm]
Tendon young modulus	$2.1*10^{11}$ [N/m^2]

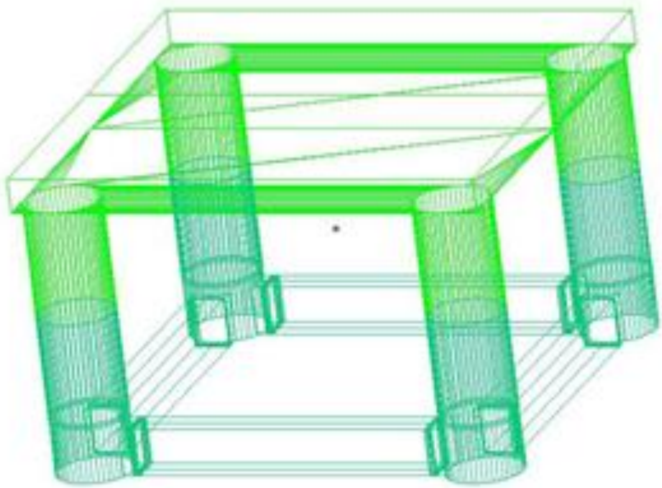

Figure 1: ISSC TLP

Table 3. Statistic for random response

Seastate	Occurrence Probability	RMS stress [MPa]	T_z
1	0.31432	2.28114	2.61E-01
2	0.27087	3.73885	5.93E-01
3	0.25149	3.741657	3.24E-01
4	0.10218	3.770279	3.49E-01
5	0.03375	4.742151	3.51E-01
6	0.00953	5.813777	2.08E-01
7	0.00239	7.486788	4.15E-01
8	0.00049	7.4988	1.24E-01

Table 4. Random variables statistics [6]

Variable	Distribution	Median / Mean	COV
Fatigue strength coefficient, A	lognormal	$\tilde{A} = 5.27 * 10^{12}$	0.63
Stress modeling error, B	lognormal	$\tilde{B} = 1$	0.2
Miner-Palmgren damage index error	lognormal	$\tilde{\Delta} = 1$	0.3
Fatigue exponent, m	constant	3	-

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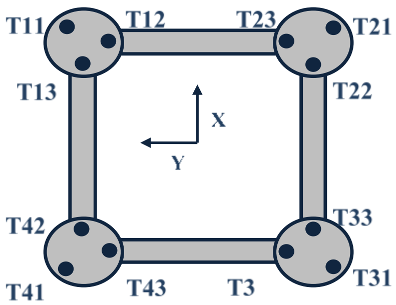

Figure 2: schematic arrangement of tendons

The system reliability analysis of TLP tendon is done assuming that it is consisted 40 elements. The results is shown in table 6. Acceptable range of reliability index based on rules is from 3.09 to 4.75 and of probability of failure from 10^{-6} to 10^{-3}. Therefore, the calculated values of probability of failure and reliability indices for joint and system are in accepted range [17].

Sensitivity analysis performed for random variables appearing in limit state function based on Miner-Palmgren damage model (Table 7). The sensitivity factors for Miner-Palmgren damage index α_Δ and fatigue strength coefficient α_A are negative hence, they are resistance variables and contribute to the resistance part of the limit state function. Sensitivity factor for stress modeling error or response uncertainty factor α_B is positive thus, it will contribute to load part of the limit state function. Therefore, an increase in Miner-Palmgren damage index and fatigue-strength coefficient will enhance the reliability of tendon. On the contrary, an increase in stress modeling error will reduce the reliability of TLP tendon. Furthermore, out of the two resistance variables, reliability is more sensitive to fatigue strength than Miner-Palmgren damage index.

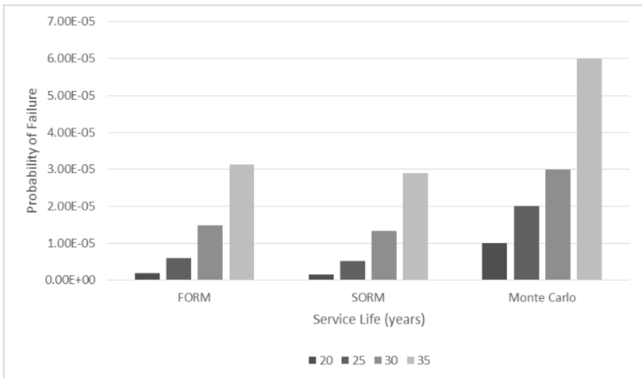

Figure 3: Effect of service life on Probability of failure

Table 5. Values of β and P_f for different service lives

Service life (years)	Monte Carlo		FORM		SORM	
	β	P_f	β	P_f	β	P_f
20	4.26489	1.00E-05	4.62554	1.87E-06	4.65572	1.61E-06
25	4.10748	2.00E-05	4.38029	5.93E-06	4.40615	5.26E-06
30	4.01281	3.00E-05	4.17645	1.48E-05	4.1987	1.34E-05
35	3.84613	6.00E-05	4.00267	3.13E-05	4.02188	2.89E-05

Table 6. Values of   and   for 20 years service-life

	β	P_f
Joint	4.62554	1.87E-06
System	3.79	7.48E-05

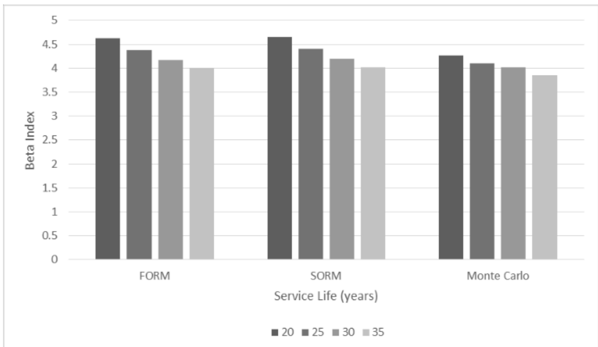

Figure 4: Effect of service life on reliability index

Table 7. Sensitivity factors

Sensitivity factor	α_Δ	α_B	α_A
Value	-0.65747	-0.33384	0.675491

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