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The Effect of Shifting Natural Frequency on the Reduction of Vortex-Induced Vibrations of Marine Risers

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ABSTRACT

Many procedures suggest for reduction of responses of riser to Vortex Induced Vibrations (VIV). Natural frequencies of marine risers is an important parameter that can affect the responses of riser to VIV. Change of riser properties such as top tension and bending stiffness can alter natural frequencies. In this study effects of riser specifications on the responses and fatigue damage of marine risers were investigated analytically and numerically. For numerically analysis 2D wake-structure coupled model is used for modeling of VIV of riser in two directions of Cross Flow (CF) and In Line (IL). The wake dynamics, including IL and CF vibrations, is represented using a pair of non-linear Van der Pol equations that solved using modified Euler method. The Palmgren–Miner Rule is used for evaluation of fatigue damage. Riser of Amir-Kabir semisubmersible placed in Caspian sea is used for case study. Because VIV is self-limiting, it is showed that lower modes have lower curvature, that in some cases this is lead to lesser stress and also fatigue damage. The results show that for tension dominant modes of vibration, natural frequencies was increased with top tension and for a certain Strouhal frequency, dominant modes of vibration was reduced which leads to reduction of stress and fatigue damage. The results show that stress and fatigue damage increased with module of elasticity of riser and reduction of this leads to reducing of stress and fatigue damage. Therefore suitable procedure for reduction of VIV responses of riser should be selected based on the current velocity.

1. Introduction

The Vortex Induced Vibration (VIV) of risers subjected to currents has been a serious concern for ocean researchers and engineers and if effects of this condition not properly mitigated it imposes high additional costs and risks on projects. When fluids flow around slender marine structures, change in the downstream pressure, caused vibration that is named Vortex Induced Vibration. When vortex shedding frequency coincide with one of the structures natural frequencies, the lock-in situation take place and riser vibrate with this natural frequency.

Reduction of VIV amplitude is a method that reduces stresses and fatigue damage which can be achieved using different ways such as increasing structural damping, avoiding resonance and manipulate the wake. Damping can be increased by various means, such as permit scraping between structural elements, use of composite materials, and materials with high internal damping, use of external dampers and etc. For avoiding resonance, the natural frequency does not coincide with the vortex shedding frequency. Many methods have been proposed in order to manipulate the wake flows behind circular cylinders and suppress VIV responses, which are classified as passive, active open-loop and active closed loop controls (Gad-el-Hak and Bushnell [1]; Choi et al. [2]; Gad-el-Hak [3]). During the past 20 years, passive control methods have been further developed, such as helical strake (Zhou et al. [4]), surface protrusions (Shih et al. [5]), shrouds (Zdravkovich [6]), splitter plate (Hwang et al. [7]), rear-wake stabilizer (Eisenlohr and Eckelmann [8]), and small rods (Zhao et al. [9]; Zhao et al. [10]). Geometric modifications of cylinder wall are the main methods for these passive controls, which inevitably increase the mass ratio and sometimes increase the drag coefficient (e.g., helical strake). Therefore, few of them are applied on a large scale in practice.

Recently some studies was performed on the effects of riser structural properties on the VIV responses. The applied top tension and also bending stiffness may significantly affect the order of CF and IL dominant harmonics, collision avoidance (among adjacent risers) and vibration suppression and can be considered as an efficient method for the control of riser statics and dynamics. Sanaati and Kato [11] present the experimental results of a study on the effects of pre-tension and axial stiffness on VIV of a horizontally mounted flexible cylinder. They showed that high pre-tension, which reduces vibration amplitude, can significantly raise the lift coefficient. They also observed that the lock-in bandwidth of amplitude response narrowed with increase in pre-tension, whereas, it broadened with axial stiffness. In addition, high applied pre-tensions delay the excitation of higher modal frequencies compared to lower pre-tensions within the same range of flow velocities. Chen et al. [12] examined effects of axially varying structural parameters, i.e. the effective tension and bending stiffness, on the dynamic characteristics and VIV of slender riser. They show that axially varying structural parameters can efficiently change the modal wave length as well as the modal displacement. The vibration amplitude of riser VIV response is influenced by the complex effect of factors involving the axial tension, bending stiffness and modal wave length. Generally speaking, for lower modes the response amplitude is larger at the axial position where the tension is smaller, whereas for higher modes the response amplitude is larger at the axial position where the bending stiffness is lower. A recent study by Lee and Gerretsen [13] showed the importance of the effect of axial stiffness on tension change and VIV motion amplitude. Lee and Allen [14] concluded that top tension and structural stiffness can have a significant impact on vibration frequencies and lock-in bandwidth. They concluded that bending-dominated cylinders have a slight increase in vibration frequency after an abrupt rise to their first lock-in frequency because the motion of the structure takes control of the shedding process. Moreover, for a tension-dominated cylinder, they concluded that there is a significant rise to the vibration frequency in each lock-in. Srinil [15] performed numerical simulations to analyze and predict the VIV of variable-tension on vertical flexible risers in linearly sheared currents. In his simulations, he emphasized the effects of tension and sheared flow on the amplitude response and multimodal VIV.

Shifting the frequencies of riser can be used to reduction of responses of riser to VIV. The present paper studied the effects of riser properties such as top tension and module of elasticity on the VIV responses of deepwater marine risers. Relation of stress and fatigue damage with these parameters is obtained analytically which can be used to better understanding of effect of shifting frequency on the VIV response of riser and selection of suitable procedure for reduction of stress and fatigue damage.

2. Formulation

Relation of stress and fatigue damage with riser structural properties (top tension and bending stiffness) obtained as follow. For a linear S-N curve in log-log scale, the expected fatigue damage per unit time can be expressed as:

$$D=\frac{n_i}{\bar{a}}\int_0^{\infty}\sigma^m f_S(\sigma)\,\mathrm{d}\sigma=\frac{n_i}{\bar{a}}E\left[\sigma^m\right] \tag{1}$$

where $\bar{a}$ and m are the scale parameter and the slope parameter of S-N curve, respectively, σ is stress, n_i is number of stress cycles at stress range, and $f_S(\sigma)$ is the Probability Density Function (PDF) for the stress cycles. The expected fatigue damage is hence directly related to the m[th] order moment, $E\left[\sigma^m\right]$ of the stress cycle PDF.

At lock-in condition response frequency will be close to vortex shedding frequency and the fatigue damage can be related directly to current velocity [16]. The damage proportionality relationship is obtained as follow:

$$D\propto f_n\sigma^m \tag{2}$$

where f_n is eigenfrequency of vibration. To obtaining the relation of stress and damage with Top Tension (*TT*) and module of elasticity (*E*) of the riser, first the mode numbers ($n_{CF,IL}$) for a tensioned string and an untensioned beam relative to *TT* and *E* are given as:

$$f_n=\begin{cases}\dfrac{n_{CF,IL}}{2}\sqrt{\dfrac{TT}{mL^2}}=\dfrac{StU}{\mathrm{Re}} \;\Rightarrow\; n_{CF,IL}=\dfrac{2StU}{\mathrm{Re}}\sqrt{\dfrac{mL^2}{TT}}\\[2ex] n_{CF,IL}^2\dfrac{\pi}{2}\sqrt{\dfrac{EI}{mL^4}}=\dfrac{StU}{\pi\,\mathrm{Re}} \;\Rightarrow\; n_{CF,IL}^2=\dfrac{2StU}{\pi\,\mathrm{Re}}\sqrt{\dfrac{mL^4}{EI}}\end{cases}$$

$$\Rightarrow n_{CF,IL}\propto 1\big/\sqrt{TT} \quad \text{:Tensioned string}$$

$$\Rightarrow n_{CF,IL}^2\propto 1\big/\sqrt{E} \quad \text{:Untensioned beam} \tag{3}$$

where *m* and *L* are mass of unit length and length of the riser , respectively, *U* is current velocity and *St* and *Re* are Strouhal and Reynoldes number, respectively. Since VIV amplitudes are self-limiting, one can assume that the response amplitudes x_0 and y_0 are independent of the current velocity and one can write:

$$\begin{cases} y(z) = y_0 \sin\left(\dfrac{n_{CF}\pi}{L} z\right) \\ x(z) = x_0 \sin\left(\dfrac{n_{IL}\pi}{L} z\right) \end{cases} \tag{4}$$

Uniaxial stresses caused by moment of each direction are obtained as:

$$\begin{cases} \sigma_{CF} = \dfrac{M_{CF} D_o}{2I} = E \dfrac{\partial^2}{\partial z^2} y \dfrac{D_o}{2} \\ \sigma_{IL} = \dfrac{M_{IL} D_o}{2I} = E \dfrac{\partial^2}{\partial z^2} x \dfrac{D_o}{2} \end{cases} \tag{5}$$

Finally, with respect to above equations, relation of stress with top tension and bending stiffness computed as follow:

$$\begin{matrix}\text{Uniaxial} \\ \text{Stress}\end{matrix} \begin{cases} \text{Tensioned String: } \sigma \propto n^2 \propto \dfrac{1}{TT} \\ \text{Untensioned Beam: } \sigma \propto E n^2 \propto \sqrt{E} \end{cases} \tag{6}$$

It can be seen that increasing of top tension lead to reduction of stress due to VIV. It is obvious that reduction of module of elasticity decrease the stress amplitude, especially in higher modes of vibration.

Using equations 2, 3, and 5 relation of fatigue damage with top tension and module of elasticity obtained as follow:

$$\begin{matrix}\text{Fatigue} \\ \text{Damage}\end{matrix} \begin{cases} \begin{matrix}\text{Tensioned} \\ \text{String}\end{matrix} : \quad D \propto f_n \sigma^m \propto \sqrt{TT}\left(n^2\right)^m \\ \begin{matrix}\text{Untensioned} \\ \text{Beam}\end{matrix} : D \propto f_n \sigma^m \propto \sqrt{E}\left(E n^2\right)^m \end{cases}$$

$$\propto \sqrt{TT}\left(\frac{1}{TT}\right)^m \propto \frac{1}{TT^{m-0.5}} \tag{7}$$

$$\propto \sqrt{E}\left(\frac{E}{\sqrt{E}}\right)^m \propto E^{0.5(m+1)}$$

Fatigue damage of riser decreased with increasing of top tension and reducing of module of elasticity. For a linear SN curve, the slope parameter m is often seen to be 3.0 [17] and so for tension dominant modes and for bending dominant modes. So for mitigation of stress and fatigue damage of VIV it is suitable that module of elasticity is reduced for higher modes (higher current velocity) and top tension increased for lower modes (lower current velocity).

3. Case study

Numerically investigation of effect of changing riser properties on the VIV responses was done using existing top tension riser of Amir-Kabir semisubmersible placed in Caspian Sea as a case study. Uniform current with various velocities was used and some results are mentioned. Based on the present position of semisubmersible, length of the riser is 713m. Other specifications of this riser are shown in the Table-1.

Table. 1 Specifications of riser of Amir-Kabir semisubmersible.

Specification	Value
Length	713 (m)
Bending stiffness (EI)	922540 (kN.m^2)
Axial stiffness (EA)	10320240 (kN)
Mass per unit length (m/L)	1080 (kg/m)
Top Tension	1000 (kN)

3.1. Modeling

There are several different available methods for prediction of VIV response of risers such as CFD, empirical models and experimental approaches. Wake oscillator model which was first introduced by Birkhoff & Zarantonello [18] used for modeling of VIV of riser. This model couples the equation of structural motion with a nonlinear oscillator equation that describes the fluid force for two directions of CF and IL. The dynamic response of a riser is described using the external force from the wake. The wake itself is described by a forced Van der Pol oscillator equation. The force term of the Van der Pol oscillator equation is related to the cylinder oscillation by a coupling term proportional to the cylinder's acceleration.

The marine riser is idealized as a tensioned Euler–Bernoulli beam. A Cartesian reference with its origin at the bottom of the riser has been used, in which the *x* axis is parallel to the flow velocity, *z* coincides with the vertical axis of the riser in its undeflected configuration and *y* is perpendicular to both as shown in the Fig. 1. A 3-D finite element model was considered for riser structure. A MATLAB code was used in this study. The Newmark-Beta method is used to solve the dynamic equation from previous section by a step-by-step time integration scheme. At each node of riser, equation of motion and Van der Pol equation of wake is coupled at each time step. Van der Pol equation is solved using modified Euler method.

The fatigue damage of a riser is evaluated based on the Palmgren–Miner Rule which Applying the Miner summation the fatigue damage is given by:

$$D = \sum_{i=1}^{N_{bin}} \frac{n_i}{N_i} \tag{8}$$

where N_i is the number of cycles to failure and n_i is the number of stress cycles at stress range $\Delta\sigma_i$ and N_{bin} is the number of stress range bins used in the Rainflow procedure.

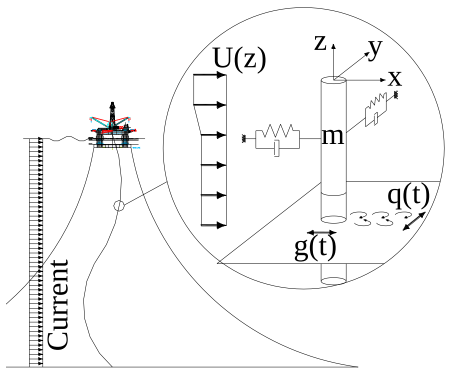

Fig. 1. 3D model of riser and 2-D wake oscillator model.

3.2 Validation of model

Chaplin et al. [19] carried out the laboratory VIV measurements of tensioned risers in a stepped current. Properties of the riser model are listed in Table 1. Lower 6m length of model was in a uniform current, while the upper part was in still water. The layout of the experiments is shown in Fig. 8. Four cases with various top tension and current velocity are chosen for VIV predictions and comparisons, as shown in Table 2.

The envelopes of CF VIV amplitude are given in Fig. 3. The present model predicts the same main dominated mode and the amplitudes are the same as experimental results. The response is mainly dominated by single mode and with increasing current velocity, the higher mode is excited. The envelopes of IL VIV amplitude are given in Fig. 4. The results of model in this direction also is the same as experiments.

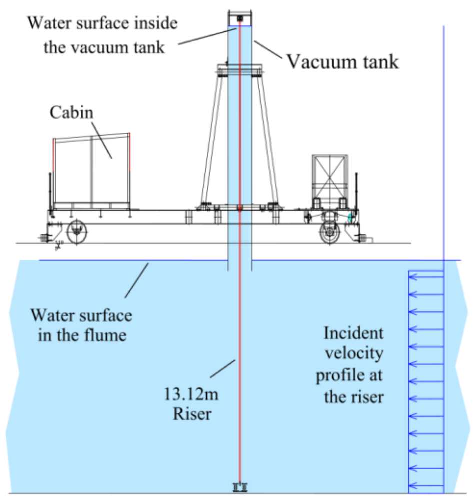

Fig. 2. Layout of Chaplin Experiments [19].

Table 1: Properties of Chaplin's tests.

Properties	Values
Total length (m)	13.12
Diameter (m)	0.028
Mass (including internal water) (kg/m)	1.85
Apparent weight (N/m)	12.1
Flexural rigidity (N/m^2)	29.9
Structural Damping	0.33%

Table 2: Test conditions for VIV comparison [19].

Cases	Top tension (N)	Current speed (m/s)
1	405	0.16
2	407	0.21
3	457	0.31
4	598	0.54
5	743	0.70

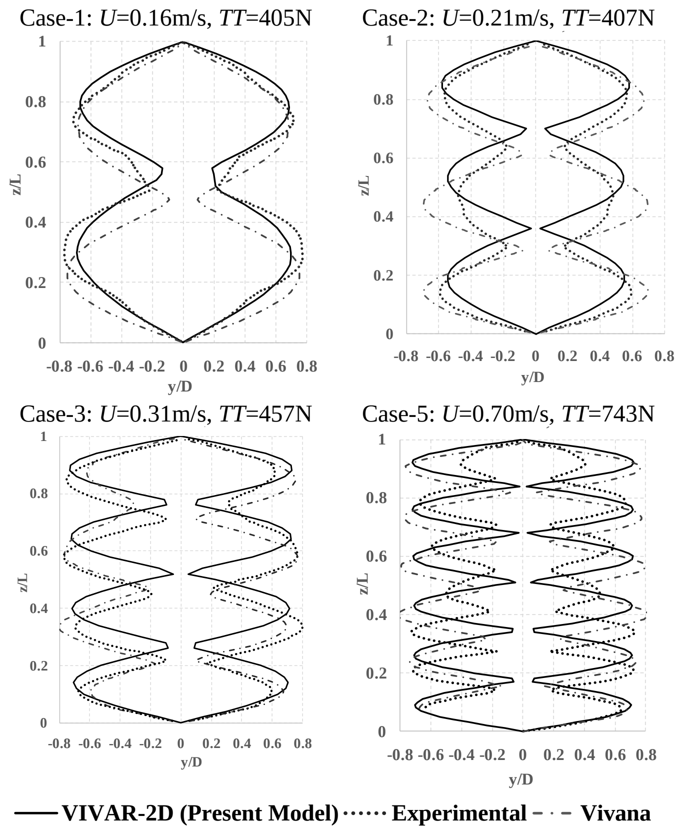

Fig. 3. Comparison of the results of the present model with others for CF VIV in stepped currents.

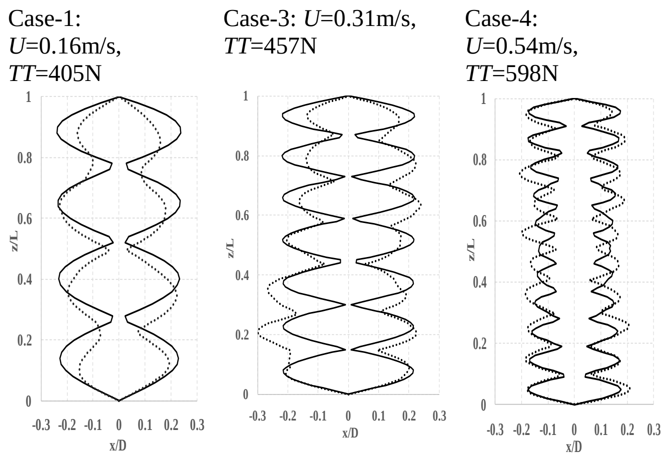

Fig. 4. Comparison of the results of present model with others for IL VIV in stepped currents.

4. Results

In this paper five cases according to Table. 2 is considered for investigation of effect of riser properties on the dynamic behavior to VIV.

Fig. 5 show natural frequencies of modes of riser and corresponding current velocity of lock-in condition for two directions of CF and IL.

Table. 2 Cases considered for VIV investigation.

Case	TT (kN)	E (kN/m^2)	Name
Case-1	1000	2.1e7	TT,E
Case-2	2000	2.1e7	2TT,E
Case-3	1000	4.2e7	TT,2E
Case-4	500	2.1e7	0.5TT,E
Case-5	1000	1.05e7	TT,0.5E

It can be seen that increasing of top tension is more effective for increasing of frequency of lower modes and increasing of bending stiffness is more effective for increasing of frequency of higher modes. It can be seen that altering top tension does not affect frequency of higher mode. This figure provides that for a certain current velocity, increasing of stiffness reduce the number of dominant modes. If the number of dominant mode reduce, the stress and fatigue damage reduce also.

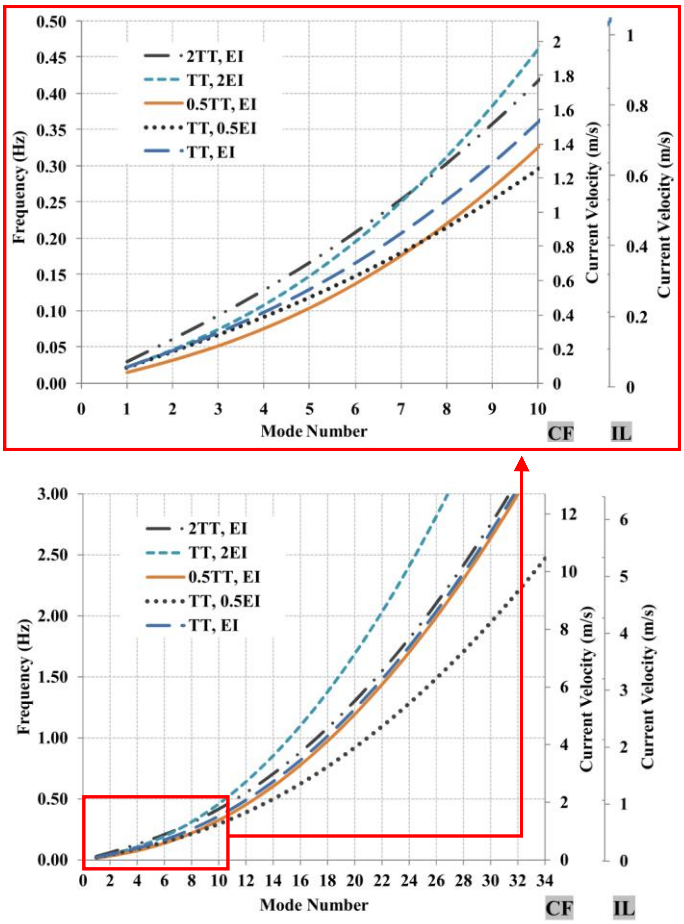

Fig. 5. Natural frequencies of riser and current velocity of lock-in conditions for two directions of CF and IL. (first 34 modes)

Fig. 6 and Fig. 7 shows Fast Fourier Transformation (FFT) of displacement of one node of riser in direction of CF for current velocities 0.2 and 1.8 m/s for five cases, respectively. This figure shows that approximately frequency of all cases is the same in IL direction but for CF direction is not. Table.3 shows frequencies of vibration of first 20 mode of riser. Also Strouhal frequency (CF direction) and twice of it (IL direction) shows in this table. In this table, frequencies that are closed to Strouhal frequency is highlighted with the same color. It can be seen that for U=0.2 m/s (tension dominant modes) in CF direction, the mode number of vibration and also frequency of vibration reduces for case with higher top tension (2*TT*,*E*) and mode number induce for lower top tension (0.5*TT*,*E*). For U=0.2 in IL direction all cases have the same frequencies at various mode numbers but mode number variation is the same as CF direction. Table 3 shows that for current velocity equals to 1.8 m/s (bending dominant mode) the mode number is reduced for cases with higher module of elasticity (*TT*,2*E*) and is induced for lower module (*TT*,0.5*E*). the same trend can be seen in IL direction in this velocity. According to table generally it is obvious that for lower velocities top tension is effective to reduce mode number of vibration and for higher velocities the bending stiffness is effective.

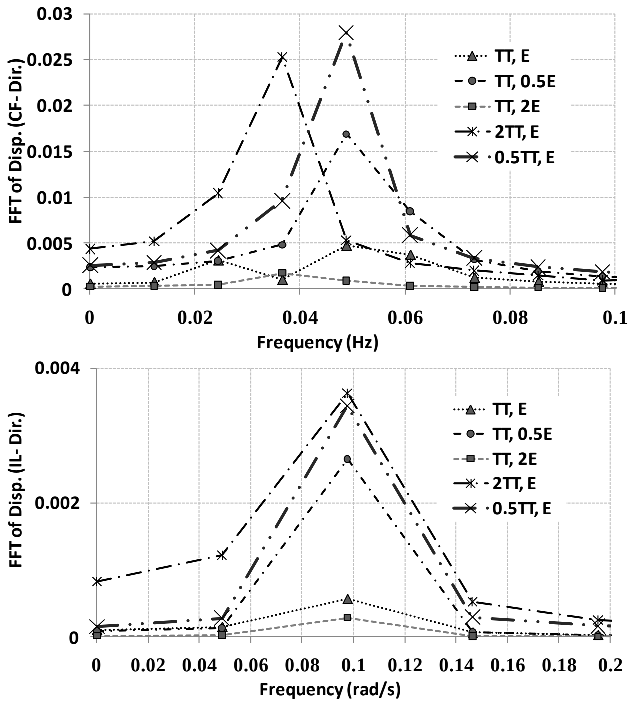

Fig. 6. Fast Fourier Transformation (FFT) of displacement of middle node of riser in CF direction-U=0.2 m/s.

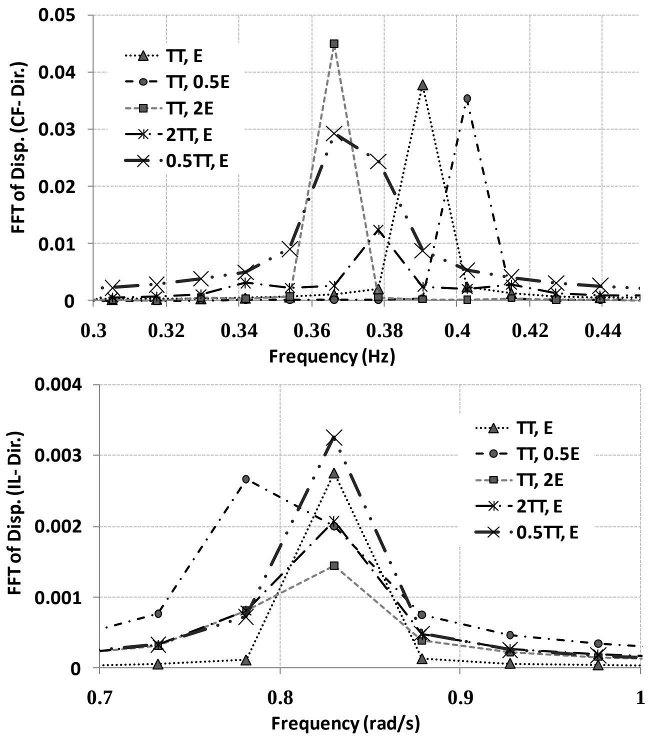

Fig. 7. Fast Fourier Transformation (FFT) of displacement of middle node of riser in CF direction-U=1.8 m/s.

Table. 3 Natural frequencies of riser and its relation with Strouhal frequency.

U (m/s)	OmegaF (Hz)	2OmegaF (Hz)
0.2	**0.047**	**0.094**
1.8	**0.424**	**0.847**

Mode Number	Frequency (Hz)				
	TT,E	TT,0.5E	TT,2E	2TT,E	0.5TT,E
1	0.018	0.018	0.019	0.026	0.013
2	0.038	0.037	0.040	0.053	0.028
3	0.060	0.058	0.066	0.082	0.046
4	0.086	0.080	0.097	0.113	0.069
5	0.115	0.104	0.135	0.147	0.096
6	0.149	0.131	0.181	0.185	0.128
7	0.188	0.161	0.234	0.228	0.165
8	0.232	0.194	0.294	0.274	0.208
9	0.281	0.231	0.363	0.326	0.256
10	0.336	0.270	0.439	0.382	0.310
11	0.396	0.314	0.523	0.444	0.370
12	0.462	0.361	0.615	0.511	0.435
13	0.533	0.412	0.715	0.583	0.505
14	0.609	0.467	0.823	0.597	0.582
15	0.691	0.526	0.844	0.661	0.663
16	0.779	0.588	1.062	0.744	0.751
17	0.872	0.725	1.194	0.832	0.844
18	0.971	0.771	1.333	0.926	0.943
19	1.076	0.799	1.481	1.025	1.047
20	1.186	0.877	1.637	1.130	1.157

Fig. 8 and Fig. 9 provide fatigue damage of riser nodes for current velocity of 0.6 m/s, for moment caused by vibration in CF and IL direction, respectively. It is evident from this figures that fatigue damage decreased with increasing of *TT* or reducing of *E*. The same trend can be seen for IL direction.

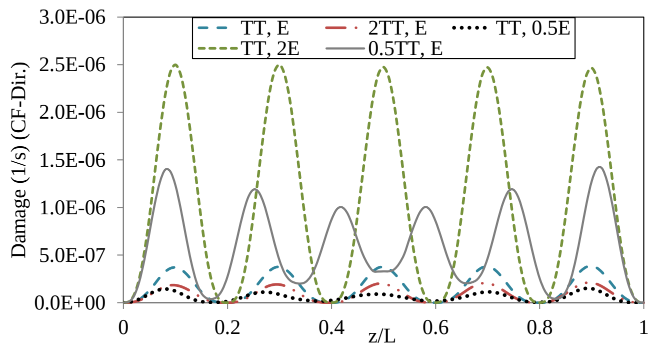

Fig. 8. Fatigue damage of riser nodes for moment caused by CF vibration for current velocity equals 0.6 m/s.

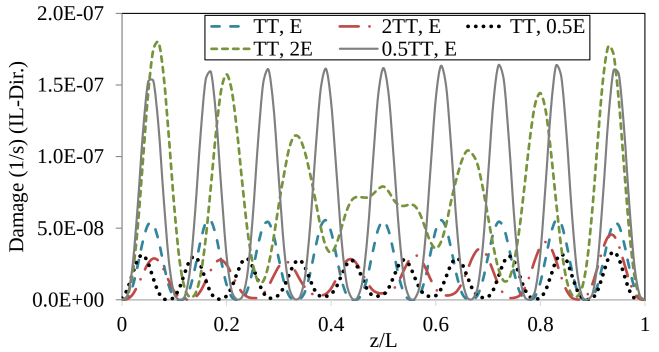

Fig. 9. Fatigue damage of riser nodes for moment caused by IL vibration for current velocity equals 0.6 m/s.

Fig. 10 and Fig. 11 show the ratio of stress due to moment of CF and IL direction for four cases (2TT, 2E, 0.5TT, 0.5E) to main structure versus current velocity, respectively. It can be seen that decreasing of *E* and increasing of *TT* cause decreasing of stress at two directions of CF and IL. According to these figures when current velocity increased, effect of TT is reduced and stress ratio of both cases of TT ratio=2 and 0.5 approach to one. Effect of variation of E approximately is constant for current velocities bigger than 0.4 m/s.

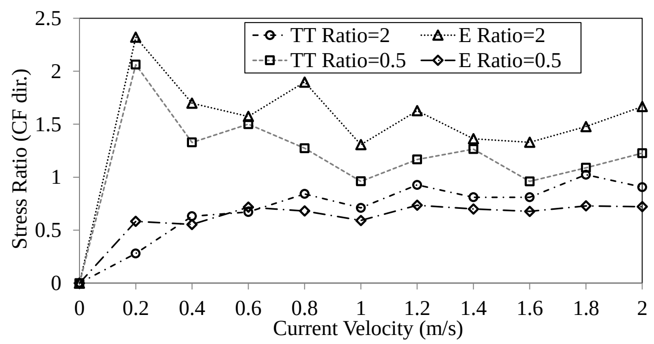

Fig. 10. Ratio of stress of moment caused by CF vibration for various cases.

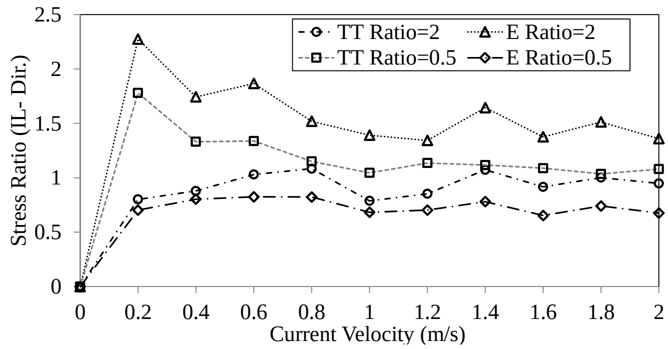

Fig. 11**. Ratio of stress of moment caused by IL vibration for various cases.**

Fig. 12 and Fig. 13 provide the ratio of fatigue damage due to CF and IL moment for four cases to main structure versus current velocity, respectively. It can be seen that increasing of TT is effective in lower velocities and for higher velocities variation of TT is not affect fatigue damage predominantly and fatigue damage ratio is very close to one for cases (TT ratio equals to 2 and 0.5). These figures shows that in low velocities, fatigue damage is very sensitive to variation of riser properties and variations of fatigue ratio is very high for various cases.

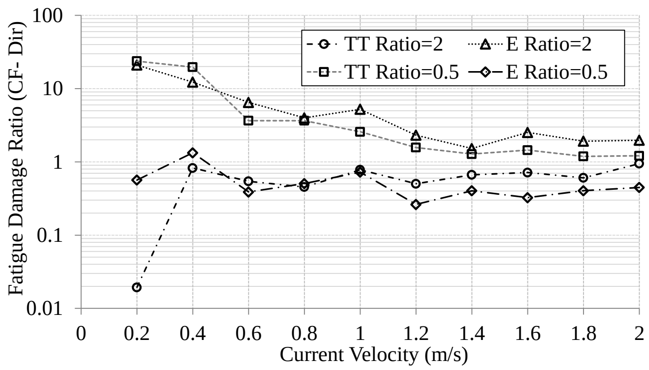

Fig. 12**. Ratio of fatigue damage of moment caused by CF vibration for various cases.**

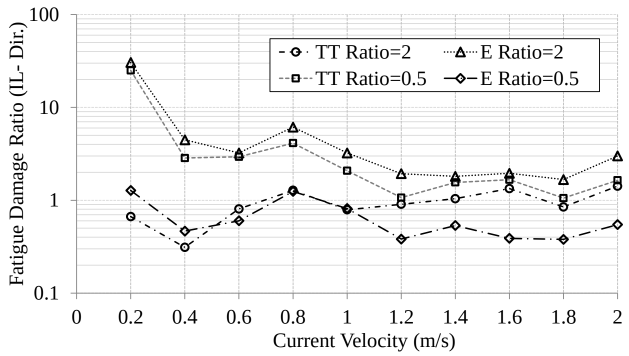

Fig. 13**. Ratio of fatigue damage of moment caused by IL vibration for various cases.**

5. Conclusion

The effects of shifting frequency on the VIV responses of marine risers were studied in this paper. Relation of stress and fatigue damage variation with riser properties such as top tension and module of elasticity is obtained. For numerical study a coupled 2D wake-structure interaction model was used for modeling of VIV that considered both direction of CF and IL. Stress of two directions was computed based on the moments that are obtained from amplitude of displacement of CF and IL. The fatigue damage of a riser is evaluated based on the Palmgren–Miner Rule and Rainflow procedure is used for counting the number of stress ranges. The following results are obtained:

- VIV has self-limiting property and for certain amplitude of vibration, higher mode of vibration has higher curvature and also stress. Also because of bigger frequency of vibration, fatigue damage increased with mode number.
- This shows that for a certain current velocity, stress for tension dominant modes is proportional to T^{-1} and for bending dominant modes is proportional to $\sqrt{E}$.
- For a certain current velocity, fatigue damage for tension dominant modes is proportional to $1/T^{(m-0.5)}$ and for bending dominant modes is proportional to $E^{0.5(m+1)}$. If the slope parameter *m* is assumed to be 3.0 [17], fatigue damage is proportional to $1/T^{2.5}$ and E^2, for tension and bending dominant modes, respectively.
- For tension-dominant modes, increasing of top tension can reduce stress and fatigue damage. This effect decreased with increasing of mode number. So this method can be used for VIV reduction for lower current velocities.
- Decreasing of module of elasticity can be used to reduce stress and fatigue damage in a wide range of current velocities.
- Increasing of bending stiffness, amplified stress and fatigue damage of riser. This is because the term of EI is used for moment computing. This effect is higher for lower modes of vibration.
- Stiffness and frequencies increased with module of elasticity, but because the *E* is used in stress relation, finally the stress and fatigue, amplified with increasing of *E*.

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