On-Bottom Stability Design of Submarine Pipelines – A Probabilistic Approach

Hadi Amlashi[1]

[1] Schlumberger, Oslo, Norway, hadi.amlashi@gmail.com

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ABSTRACT

Un-trenched submarine pipelines will experience the wave and current loads during their design lifetime which potentially tend to destabilize the pipeline both horizontally and vertically. These forces are resisted by the interaction of the pipe with the surrounding soil. Due to the uncertainties involved in the waves, currents and soil conditions, there will be a complex interaction between the wave/current, pipeline and seabed that needs to be properly accounted for. The design of submarine pipelines against excessive displacements due to hydrodynamic loads (DNV-RP-F109) is defined as a Serviceability Limit State (SLS) with the target safety levels as given in DNV-OS-F101 (2013). In this paper, uncertainties associated with the on-bottom stability design of submarine pipelines are investigated. Monte Carlo Simulations (MCS) are performed as the basis for the probabilistic assessment of the lateral stability of the pipeline located on the seabed. Application of the method is illustrated through case studies varying several design parameters to illustrate the importance of each design parameter for exceeding a given threshold of the SLS criterion. Uncertainties in the significant wave height and spectral peak period are found to be important parameters in describing the Utilization Ratio (UR) distribution. Type of the soil has also an impact on the distribution of UR, i.e. how the passive soil resistance in the pipe-soil interaction model is accounted for. Therefore, the definition of characteristic values of both loads and resistance variables are important for the UR.

1. Introduction

Offshore pipelines have long since been an efficient means of transport for oil and gas. Submarine pipelines as installed upon seabed are subject to waves and currents. Moreover, uncertain or unknown soil conditions are a common cause of construction delays and cost escalations for submarine pipeline projects. There exists a complex interaction between waves/currents, the pipeline, and the seabed that needs to be properly accounted for.

To mitigate the lateral instability of a pipeline left exposed on the seabed, either the pipeline should be stabilized using an appropriate concrete weight coating or a thicker wall thickness, or be anchored/trenched locally. Both methodologies are expensive and complicated from design and construction/installation point of view.

Several studies have been performed to investigate the major physical phenomena involved in predicting the lateral stability of un-trenched pipelines on the sea floor. Among, was the Pipeline Stability Design (PIPESTAB) JIP [1] which included both analytical and experimental investigations to arrive at a technically sound basis for the on-bottom lateral stability design of submarine pipelines [2,3]. Other researches performed are: (1) the AGA project [4], and (2) a research project at the Danish Hydraulic Institute (DHI) [5]. Based on these researches, several pipe-soil interaction models were introduced.

In a typical pipe–soil interaction model, the total soil lateral resistance to pipeline movement, F_H, is assumed to be the sum of the sliding (friction) resistance component (F_f) and the soil passive resistance component (F_R), i.e.

$$F_H = F_f + F_R = \mu(W_s - F_L) + F_R \qquad (1)$$

where μ = friction (resistance) coefficient; W_s = pipeline submerged weight per meter, F_L = lift force upon pipeline per meter and F_R = soil passive resistance per meter which depends on the soil buoyant weight and the contact area between the pipeline and the soil. The lateral soil resistance (F_H) should balance the designed wave/current loads upon the pipeline, which can be calculated with the wake model proposed by [6] considering the oscillatory flow over the pipeline.

The outcome of the above-mentioned researches has been the basis for further development of DNV-RP-F109 (2011) [7]. For the lateral on-bottom stability, three different design methods are described in DNV-RP-F109 (2011):

1) Dynamic lateral stability analysis
The dynamic lateral stability analysis is based on a time domain simulation of the pipe response, including hydrodynamic loads from an irregular sea-state, soil resistance forces, boundary conditions and the dynamic response of pipeline. Usually, the dynamic analysis forms the basis for the validation of other simplified methods such as the generalized analysis method. Therefore, it is normally used for the detailed analysis of critical areas along a pipeline, such as pipeline crossings, riser connections etc., when the uncertainties in the design parameters calls for a detailed assessment.

2) Absolute lateral static stability method
An absolute static requirement for the lateral on-bottom stability of pipelines is based on the static equilibrium of forces that ensures the resistance of the pipe against motion is sufficient to withstand maximum hydrodynamic loads during a sea state, i.e. the pipe will experience no lateral displacement under the design extreme single wave-induced oscillatory cycle in the sea state considered. One should, however, note that this approach does not account for the increased passive resistance that is built up due to the pipeline penetration caused by the wave-induced flow. The absolute stability method may be relevant for e.g. pipe spools, pipes on narrow supports, cases dominated by current and/or on stiff clay.

3) Generalized lateral stability method
The generalized lateral stability method is based on an allowable displacement in a design spectrum of oscillatory wave-induced velocities perpendicular to the pipeline at the pipeline level. This can be performed for No-Break Out (NBO), i.e. displacement < 0.5 diameter or a multiplier of pipeline diameter (limited by 10).

The soil behaviour in each application is not always in accordance with its soil classification. This is particularly true when the particle size distribution falls near the classification boundary of coarse/fine soils and soil classification alone may not fully capture the soil behaviour for aspects of design and operation [8]. Therefore, proper knowledge of how the soil classification is carried out and its limitations, is required in order to use the geotechnical survey data correctly and efficiently. This is particularly important when performing the on-bottom lateral stability analysis of a pipeline subject to waves and currents. This is because the interaction between waves, currents and soils and their corresponding

uncertainties are important for correctly accounting of the passive soil resistance.

From the design point of view, the stability of submarine pipelines against excessive displacement due to hydrodynamic loads is normally ensured by the use of a Load and Resistance Factors Design Format (LRFD), as given e.g. in DNV-RP-F109 (2011). The excessive lateral displacement due to the action of hydrodynamic loads is defined as a Serviceability Limit State (SLS) with the target safety levels given in DNV-OS-F101 (2013) [9]. If this displacement leads to significant strains and stresses in the pipe itself, these load effects should be dealt with in accordance with relevant codes, e.g. DNV-OS-F101 (2013). Generally, SLS criterion is a condition which, if exceeded, renders the pipeline unsuitable for normal operations. In DNV-OS-F101 (2013), exceedance of a SLS category are evaluated as an Accidental Limit State (ALS).

To document how the variability in hydrodynamic loads and the soil behaviour and their interactions are accounted for in the traditional design practices, a probabilistic approach is therefore introduced.

In this paper Monte Carlo Simulations (MCS) are performed as the basis for the probabilistic assessment of the lateral stability of the pipeline located on the seabed. Application of the method is illustrated through case studies to illustrate the importance of each design parameter for exceeding a given threshold of the SLS criterion.

2. On-bottom Stability - Design Methodology
The on-bottom stability design of submarine pipelines is normally performed using DNV-RP-F109 (2011) together with DNV-OS-F101 (2013). Design methods and acceptance criteria for vertical and lateral stability of pipelines on the seabed are briefly explained below.

Vertical Stability
For the vertical stability, a simple design equation, based on sinking in the sea water, is presented. The criterion is defined based on a single safety factor on the total weight per unit length as bellow:

$$W_{dry} \geq \gamma_w b \qquad (2)$$

or

$$s_g \geq \gamma_w \qquad (3)$$

where s_g is the pipe specific density defined as $1 + W_s/b$ and γ_w is the weight safety factor. Normally, a safety factor of $\gamma_w = 1.1$ is used. The W_s is the submerged weight of the pipeline defined as $W_s = W_{dry} - b$, the buoyancy $b = \rho_w g \pi D^2 / 4$ and D is the outer diameter including all coatings. The dry weight of the pipeline (W_{dry}) reads:

$$W_{dry} = W_{steel} + W_{coating} + W_{content} \qquad (4)$$

Lateral Stability

For the lateral stability, both Coulomb friction resistance and the passive resistance from soil are accounted for. A general design criterion is presented as below:

$$Y(X) \leq Y_{allowable} \tag{5}$$

where Y is the non-dimensional lateral pipe displacement ($Y = y/D$), the vector X contains main design parameters influencing the accumulated displacement (y) and $Y_{allowable}$ is the allowed non-dimensional lateral displacement (scaled to the pipe diameter). The $Y_{allowable}$ shall be defined based on design conditions, but generally the total accumulated displacement is limited to 10-pipe diameter.

For the lateral on-bottom stability, three different design methods are presented:

1) A dynamic lateral stability analysis
2) An absolute lateral static stability method.
3) A generalized lateral stability method based on database results from dynamic analyses

These methods are briefly addressed and the uncertainties involved are discussed subsequently.

Dynamic lateral stability analysis

The most complete approach for on-bottom lateral stability of a pipeline is to perform a dynamic simulation.

The dynamic response of a pipeline depends on wave to current ratio, wave period and soil type and penetration (passive resistance). Due to high nonlinearities involved in the response of the pipeline, a complete sea state should be used. In lack of proper full sea state data, a 3-hours sea state data shall be used.

Time-domain simulations are performed to calculate the accumulated lateral displacement of a pipeline subjected to hydrodynamic loads from waves, currents and soil resistance forces.

A typical result from the dynamic on-bottom stability analyses is shown in **Figure 1**. An envelope curve is typically established based on many analysis cases to calculate the maximum allowable displacement. Time increments should be small enough to capture the actual nonlinear behaviour of the pipe-soil interaction. Furthermore, the axial force (due to internal pressure and temperature) and end effects should properly be accounted for. Also, the wave directionality together with the maximum wave height and the sequence/number of waves must be accounted for. Hence, many analyses should be performed to establish a representative envelope curve. Normally, a parametric study is performed to cover a displacement ranges of zero to several diameters of the pipeline.

It should be noted that the $Y_{allowable}$ may further be limited due to the excessive bending of the pipeline and other constraints.

This approach is very time consuming and requires the design data to be available, i.e. in later design phases (detail design) where optimization of the pipeline weight may be possible due to the availability of site-specific environmental data.
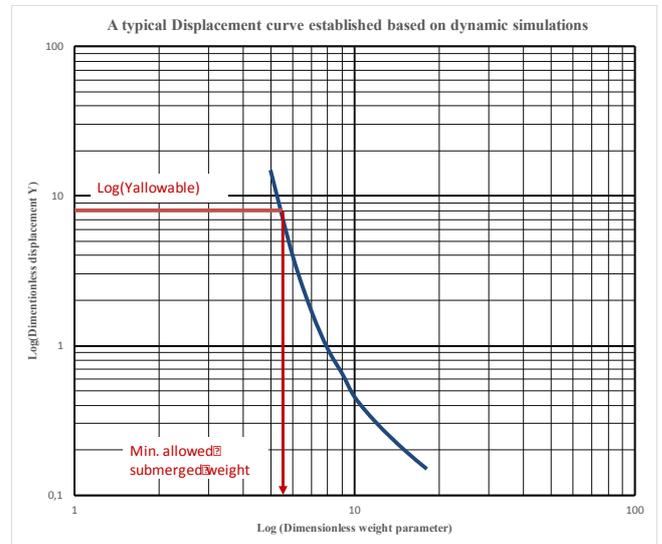

Figure 1 A typical Normalized Displacement versus Normalized Weight (typically shown for results from dynamic analyses)

However, in earlier phases of design, e.g. feasibility and/or conceptual design stages, simpler yet reliable design approaches are required to ensure the on-bottom stability of the pipeline accounting for the uncertainties in environmental loads and the soil behaviour.

Absolute lateral stability method

The absolute lateral stability criteria, in DNV-RP-F109 (2011), are defined in terms of UR (Utilisation Ratio) as described below:

$$UR_{AS,1} = \gamma_{SC} \frac{\mu F_Z^* + F_Y^*}{\mu W_s + F_R} \leq 1 \tag{6}$$

$$UR_{AS,2} = \gamma_{SC} \frac{F_Z^*}{W_s} \leq 1 \tag{7}$$

where, γ_{SC}= Safety factor for Safety Class and μ= Friction coefficient. The design Peak load coefficients (C_z^* and C_y^*), ref. to Eqs. (13) and (14) and are calculated from design tables in DNV-RP-F109 (2011) by interpolations of design parameters Keulegan-Carpenter number ($K^* = U^* T^*/D$) and steady to oscillatory velocity ratio ($M^* = V^*/U^*$) for a single design oscillation.

The above criteria can be stated in terms of minimum required submerged weight ($W_{s,req.}$). It means, the criterion can read:

$$\frac{W_s}{W_{s,req.}} \geq 1 \tag{8}$$

The relation between the Utilisation Ratio (UR) and the minimum required submerged weight ($W_{s,req.}$) for

both lateral and vertical stabilities, in Eqs. (6) and (7) , can then be written as follow:

$$UR_{AS,1} = \frac{\mu W_{s,lat.req.} + F_R}{\mu W_s + F_R} \leq 1 \qquad (9)$$

$$UR_{AS,2} = \frac{W_{s,ver.req.}}{W_s} \leq 1 \qquad (10)$$

where $W_{s,lat.req.}$ and $W_{s,ver.req.}$ are:

$$W_{s,lat.req.} = \frac{\gamma_{SC}(F_Y^* + \mu F_Z^*) - F_R}{\mu} \qquad (11)$$

$$W_{s,ver.req.} = \gamma_{SC} F_Z^* \qquad (12)$$

The peak vertical load (F_Z^*) and peak horizontal load (F_Y^*) are defined as:

$$F_Z^* = 0.5 r_{tot,z} \rho_w D C_Z^* (U^* + V^*)^2 \qquad (13)$$

$$F_Y^* = 0.5 r_{tot,y} \rho_w D C_y^* (U^* + V^*)^2 \qquad (14)$$

By introducing equations (13) and (14) into (11) and (12), the following criteria are derived:

$$W_{s,lat.req.} =$$
$$\left[\frac{\gamma_{SC}\left(\frac{r_{tot,y}C_Y^* + \mu r_{tot,z}C_Z^*}{L^*}\right) - f_{passive}\left(1 - \frac{r_{tot,z}C_Z^*}{L^*}\right)}{\mu} \right] W_s \qquad (15)$$

$$W_{s,ver.req.} = \left[\gamma_{SC}\left(\frac{r_{tot,z}C_Z^*}{L^*}\right)\right] W_s \qquad (16)$$

The parameters are as defined in Table 1.

Table 1 Definition of Design Parameters

Characteristic	Value
L = Significant weight parameter for virtually stable pipe	$W_s / 0.5 \rho_w D U_s^2$
L^* = Weight parameter related to single design oscillation	$W_s / 0.5 \rho_w D (U^* + V^*)^2$
$r_{tot,z}$ = Vertical load reduction due to pipe-soil interaction	$0.7\left(1 - 1.3\left(z_p/D - 0.1\right)\right)(1 - 0.14(\theta_t - 5)^{0.43})(z_t/D)^{0.46}$
$r_{tot,y}$ = Horizontal load reduction due to pipe-soil interaction	$(1 - 1.4z_p/D)(1 - 0.18(\theta_t - 5)^{0.25})(z_t/D)^{0.42}$
U^* = Oscillatory velocity amplitude for single design oscillation, perpendicular to pipeline	$0.5 U_s\left(\sqrt{2ln\tau} + 0.5772/\sqrt{2ln\tau}\right)$
$V^* = V$ = Steady current velocity perpendicular to pipeline	$U_r\left(\frac{(1 + z_0/D)ln(1 + D/z_0) - 1}{ln(1 + z_r/z_0)}\right)\sin\theta_c$
U_s = Spectrally derived oscillatory velocity (significant amplitude) for design spectrum, perpendicular to pipeline	$\frac{H_s}{T_n} \cdot f(T_n/T_p)$ $f(T_n/T_p)$, based on linear wave theory, can be graphically derived from Fig. 3-2 in DNV-RP-F109 (2011)
T_n = Reference period	$\sqrt{d/g}$
τ = Number of oscillations in the design bottom velocity spectrum	T/T_u T = Design duration of sea states (normally a 3-hour duration is recommended)
T_u = Spectrally derived mean zero up-crossing period	$T_p \cdot g(T_n/T_p)$ $g(T_n/T_p)$, based on linear wave theory, can be graphically derived from Fig. 3-3 in DNV-RP-F109 (2011)
$f_{passive}$	Passive resistance factor which depends on several parameters such as: • pipe penetration into soil (z_p) • clay strength parameter ($G_c = s_u/D\gamma_s$) or • sand density parameter ($G_s = \gamma_{sw}/g\rho_w$), etc. $f_{passive}$ can be calculated from Eqs. 3.23-3.29 in DNV-RP-F109 (2011).
H_s = Significant wave height during a sea state	Design Input
T_p = Design Spectral Peak Period	Design Input
U_r = Design Steady current velocity near seabed	Design Input
γ_s = Dry unit soil weight for Clay	Design Input
S_u = Un-drained clay shear strength	Design Input
γ_{sw} = Submerged unit soil weight for Sand	Design Input
μ = Soil friction factor	Design Input

z_t and θ_t are the trench depth and the trench angle, while z_p is the soil penetration depth. For the definition of other parameters, reference is made to DNV-RP-F109 (2011).

The characteristic load condition considered reflects the most probable extreme response during either a temporary phase (less than 12 months), i.e. an installation phase or a permanent operational condition (excess of 12 months), i.e. a production phase.

Generalized lateral stability method

The generalized lateral stability criteria, in DNV-RP-F109 (2011), can be defined in terms of UR as below:

$$UR_{GS,NBO} = \frac{W_{s,NBO,all.}}{W_s} \leq 1 \tag{17}$$

$$UR_{GS,Acc.} = \frac{W_{s,Acc.,all.}}{W_s} \leq 1 \tag{18}$$

The allowable weight for No-Break Out ($W_{s,NBO,all.}$) and the allowable accumulated weight ($W_{s,Acc.,all.}$), corresponding to a displacement of $y = Y \cdot D$, are calculated as below:

$$W_{s,NBO,all.} = 0.5\rho_w D U_s L_{stable} \tag{19}$$

$$W_{s,Acc.,all.} = 0.5\rho_w D U_s L_Y \tag{20}$$

The significant weight parameter (L), in general, is defined as:

$$L = W_s / 0.5\rho_w D U_s^2 \tag{21}$$

L_{stable} and L_Y are significant weight parameter for virtually stable pipe (e.g. within $0.5D$) and displacement of up to 10-pipe diameter, respectively, and are determined by interpolation of design curves given in DNV-RP-F109 (2011) using three non-dimensional parameters, i.e. M (Steady to oscillatory velocity ratio for design spectrum), N (Spectral acceleration factor) and K (Significant Keulegan-Carpenter number). The parameters are defined as follows:

$$M = V/U_s \tag{22}$$

$$N = (U_s/T_u)/g \tag{23}$$

$$K = U_s T_u / D \tag{24}$$

The interpolation can be performed for both pipe on soil and on clay, however with some limitations.

It should, however, be noted that only one check needs to be passed to ensure that the pipeline is laterally stable, i.e. either the absolute stability or the generalized stability. However, in most cases of designs against the on-bottom lateral stability, the following relationship may exists:

$$W_{s,all.} \leq W_{s,req.} \leq W_s \tag{25}$$

where $W_{s,req.} = \max(W_{s,ver,req.}, W_{s,lat,req.})$ and $W_{s,all.}$ is either $W_{s,NBO,all.}$ or $W_{s,Acc.,all.}$, depending on the allowed pipe displacement.

Therefore, a certain level of interpretation should be applied to determine what is deemed to be a stable pipeline with given viabilities in design parameters. This will be discussed subsequently in this paper.

3. Reliability Basis
General

The adequate structural safety of offshore pipelines is ensured by design, load/response monitoring and inspection during their design life.

Reliability methods are now widely used to make optimal decisions regarding safety and life cycle costs of offshore structures, as mentioned e.g. in [10] and [11].

Such methods can deal with uncertainties associated with the design, fabrication, installation and operation of e.g. pipeline systems, and may be classified as follows:

  a) Structural Reliability Analysis (SRA), see e.g. [12]. The purpose of SRA is to determine the failure probability considering fundamental variability, and natural and man-made uncertainties due to lack of knowledge.
  b) Quantitative Risk Analysis (QRA) which deals with the estimation of likelihood of fatalities, environmental damage or loss of assets in the broad sense.

The focus in the present paper is on the first one, i.e. Structural Reliability Analysis (SRA).

For the considered failure mode, the possible realizations of $\boldsymbol{X}$ (a vector of n random variables) can be separated in two separate domains; namely the safe domain and the failure domain. The curved surface between the safe and failure domains in the space of basic variables is denoted as the limit state surface, and the reliability problem is conveniently described by a so-called limit state function $g(\boldsymbol{X})$.

The probability of failure is the probability of occurrence in the failure domain:

$$P_f = P[g(\boldsymbol{X}) \leq 0] = \iint_{g(\boldsymbol{X}) \leq 0} f_X(\boldsymbol{x}) d_x \tag{26}$$

Where $f_X(\boldsymbol{x})$ represents the joint probability density function for $\boldsymbol{X}$ and represent the uncertainty in the governing random variables. The integral of equation above may be calculated by direct integration, simulation or FORM/SORM methods. In general, the accuracy of the method should be validated before use. Unless it is not done earlier for the problem in hand, crude Monte Carlo simulations should be used to validate other approximate methods.

The "tail sensitivity problem" causes the computed failure probability to be of limited informative value except for reliability comparisons made in the same

model space of probability distributions as mentioned e.g. in [12]. The target level needs to be determined based on the same reliability methodology that will be applied to demonstrate compliance with the target level, see e.g. [10].

The reliability index can be defined to express the safety defined in the space of random variables, as $\beta = \Phi^{-1}(P_f)$, see [13].

For the simple case, where X consists of two variables, i.e. the load L and the resistance R, the limit state function can be specified as $g(X) = R - L$ with the distribution and the characteristic values as shown in **Figure 2** (DNV-RP-C207, 2012) [14].
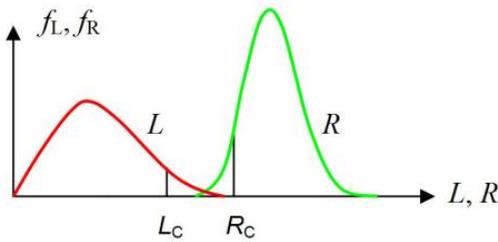

Figure 2 A typical Reliability load and strength curves together with the corresponding characteristic values

In the partial safety factor method, the design resistance R_d should be larger than the design load effect L_d for the structural elements with which are verified for several different load combinations. The design load and resistance are related to the characteristic values as follow:

$$L_d = \gamma_L L_c \quad \& \quad R_d = \gamma_R R_c \qquad (27)$$

The characteristic values are defined as a quantity associated with the probability distribution for loads and resistance variables. This is further discussed in the following section.

Number all tables and figures according to their appearance.

4. Probabilistic on-bottom lateral stability
Methodology

The cumulative distribution function F_X of a random variable X is defined as the probability that X falls short of x:

$$F_X(x) = P[X \leq x] \qquad (28)$$

where P[.] denotes probability. The probability of exceedance Q_X is defined as the complement of the cumulative distribution function:

$$Q_X(x) = 1 - F_X(x) = P[X > x] \qquad (29)$$

The p quantile in the distribution of X is the value of X whose cumulative distribution function value is p, as defined below:

$$F_X(x_p) = p \qquad (30)$$

Both the generalized and the absolute lateral stability methods presented above are applied probabilistically accounting for the random variables stated in the proceeding chapter.

In reliability-based limit states design of pipelines, it is assumed that the loads and resistances follow some assumed distributions. However, it is important to accurately model the on-bottom stability distribution and particularly its tail behaviour. Moreover, both the systematic (bias) and random model uncertainties need to be addressed. Monte Carlo Simulation techniques can be applied for this purpose. The advantage of the MCS method is that it converges towards exact results when enough simulations are carried out. A drawback is that it is time-consuming especially if small failure probabilities are to be estimated. A comprehensive overview of computational methods for the probability measures can be found in Melchers (1999). In the present study, MCS method is used for comparison purpose.

Monte Carlo Simulation (MCS) approach in Excel2013 is used for this matter. A VBA code is written to perform the simulations while assigning any number of random variables with their associated uncertainties.

In the present work, the criteria for UR_{AS} and UR_{GS} are used in probabilistic analyses. The suitability of the method is benchmarked against common design approach, i.e. DNV-RP-F109 (2011).

Uncertainty measures

Uncertainties associated with random variables have many sources, but in general, may be categorized as two main types of uncertainty, see e.g. Madsen, H.O. et al. (1986):

    – Aleatory, i.e. physical uncertainty

    – Epistemic, i.e. uncertainty related to imperfect knowledge.

Physical uncertainty $(\hat{\chi}_p)$ is a natural randomness of a quantity which cannot be reduced, e.g. the random variability in the soil strength from a point measurement within a soil sample.

Uncertainty due to the imperfect knowledge, however, consists of statistical uncertainty, model uncertainty and measurement uncertainty which can, in principle, be reduced by the collection of more data, by improving engineering models and by employing more accurate methods of measurement:

a) The statistical uncertainty $(\hat{\chi}_{st})$ is caused by limited number of observations of a random quantity.

b) Te model uncertainty $(\hat{\chi}_m)$ is caused by idealized engineering models used for the representation and prediction of quantities such as the passive soil resistance. The model uncertainty involves two elements, viz. (1) a bias (*B_mod*) if the model systematically leads to over-prediction or under-prediction of a quantity in question and (2) a randomness

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($\hat{\chi}_r$) associated with the variability in the predictions from one prediction of that quantity to another.

c) The measurement uncertainty ($\hat{\chi}_{ms}$) is caused by imperfect instruments and sample disturbance when a quantity is observed. Like the model uncertainty, the measurement uncertainty involves two separate elements, i.e. the systematic bias and the random error.

It is noted that uncertainties due to the imperfect knowledge are statistically independent of physical (natural) uncertainties. The above stated uncertainties, are all represented by their generic distribution types and associated distribution parameters.

The measurement uncertainty can, conservatively, be disregarded regardless of the accuracy of the method of the measurement, as this is either unknown or is very difficult to quantify. This is because the physical uncertainty estimate ($\hat{\chi}_p$) implicitly account for the measurement uncertainty with the given accuracy of the measurement. The net physical uncertainty estimate ($\hat{\chi}_{p,net}$) should then be represented as:

$$V_{\hat{\chi}_{p,net}} \approx \sqrt{V_{\hat{\chi}_p}^2 - V_{\hat{\chi}_{ms}}^2} \qquad (31)$$

where V denotes the coefficient of variation. Therefore, only physical, stochastic and model uncertainties need to be accounted for. The total uncertainty can then be formulated as, Ref. [12]:

$$\hat{\chi}_{tot} = \hat{\chi}_p \hat{\chi}_{st} B_{mod} \hat{\chi}_r \qquad (32)$$

The determination of uncertainties in random variables for hydrodynamic loads and the soil resistance is a cumbersome task to perform and requires sufficient statistical data. Since the design methods given in DNV-RP-F109 (2011) are used here, it is assumed that these uncertainties are properly accounted for in the given design formats. However, definition of characteristic design values e.g. parameters associated with hydrodynamic loads and the soil resistance is subject to uncertainty and should properly be accounted for.

Due to lack of the readily available statistical data, the assumed random variables with associated uncertainties for the considered characteristic design values in case studies, are further discussed in the proceeding chapter.

Characteristic values
For practical deterministic design by codes and standards (such as DNV-RP-F109), a characteristic value is rather used instead of entire variability associated with the specified probability distribution. This is usually defined as a (characteristic) quantity associated with the assumed probability distribution. Examples of such quantity are:
1) The mean value

2) A quantile in the probability distribution, e.g. the 5% (or 95%) quantile
3) The mean value (plus) minus a factor of standard deviations, i.e. $\mu \pm k\sigma$. For example, $\mu - 2\sigma$ (for a normal distribution) corresponds to the 2.3% lower quantile.
4) The most probable value, i.e. the value for which the probability density function is maximum.

The choice of the characteristic value usually depends on the design code, e.g. the choice of confidence and on the actual application, e.g. design constraints. In general, due to uncertainties involved in a random variable described by a probability distribution, the estimated characteristic value also becomes statistically uncertain. To properly account for such a statistical variability, it is common to specify the characteristic value with a specified confidence level. An adequate confidence should, therefore, be used for the estimation of the characteristic value from the data. This is not explicitly defined in DNV-RP-F109 (2011) and therefore is subject to the understanding of the user.

In this paper, due to lack of proper statistical data, a characteristic value of $\mu \pm 1.5\sigma$ is assumed for loads and resistance variables. This is equivalent to approximately 6% (94%) lower (upper) quantile. It is emphasized that this assumption is also subject to uncertainty, but used in this paper to benchmark the effect of this definition on the probabilistic analysis.

Random variables and uncertainties
Table 2 and **Table 3**, respectively, define the main parameters for two hypothetical design cases, i.e. case (1) in water depth of 330m and clay soil type and case (2) in water depth of 135m with sand soil types.

Due to lack of proper statistical data, normal distribution (with a bias of 1.0) is assumed for all random variables. This is subject to uncertainty, but it is assumed here for the sake of simplicity and to benchmark the effect of variables randomness on the results.

The CoV of 0.15 is used for all load variables. For resistance variables, the CoV of soil unit weight variables (dry and submerged) are assumed to be 0.1, while the two other variables, i.e. the undrained shear strength and the friction coefficient are assumed to have a CoV of 0.15. The choice of random uncertainty (CoV) is arbitrary in these examples. However, as a general practice, more uncertainties are given to load variables than resistance variables.

Case (1)
The water depth in Case (1) is 330m. The soil type is assumed to be clay. Case (1) is more relevant for the absolute lateral stability design criteria in which the soil passive resistance, due to less interaction of

waves and the seabed, is negligible as compared to the sliding resistance, see Eq(1). The deterministic analysis using the design wave and current return period values is calculated as a reference to compare it later with the probabilistic results.

The operational load case with the 100yrs wave combined with 10yrs current loads (Normal safety class with γ_{SC} of 1.4 for the pipeline in clay and North Sea winter storm) , see DNV-RP-F109 (2011), will give the worst load combination.

The results are as follows:

$UR_{AS,1} = 0.993$,

$UR_{AS,2} = 0.229$,

$UR_{GS,NBO} = 3.402$ and

$UR_{GS,Acc.} = 0.427$.

The predicted displacement (y), associated with the generalized accumulated displacement approach, is 1.51m ($\approx 2.9D$).

As expected, the NBO criterion gives an unrealistically high UR. This is because at deep waters, K may be low (2.3) while M, due to the presence of current, will be very high (7.9) and therefore the result will be outside of validity range of the required stable weight for NBO, i.e. $4 \leq K \leq 40$ and $0 \leq M \leq 0.8$.

Case (2)

The water depth in Case (2) is 135m. The soil type is assumed to be coarse sand. Case (2) in Table 3 is more prone to the wave-soil interaction and hence the passive soil resistance will play an important role in total soil resistance. Therefore, the absolute lateral stability for the design becomes very conservative and hence the generalized lateral stability design criteria suits better.

For the pipeline in Case (2), the operational load case with the 100yrs wave combined with 10yrs current loads (Normal safety class with γ_{SC} of 1.32 for the pipeline in sand and North Sea winter storm), see DNV-RP-F109 (2011), will give the worst load combination.

The results are as follows:

$UR_{AS,1} = 2.347$,

$UR_{AS,2} = 0.778$,

$UR_{GS,NBO} = 1.579$ and

$UR_{GS,Acc.} = 0.769$.

The predicted displacement (y), associated with generalized accumulated displacement approach, is 1.82m ($\approx 3.4D$).

As expected, the AS criterion gives high conservative UR, while the accumulated generalized stability (accounting for the passive soil resistance) will satisfy the design. It is, however, noted that the NBO criterion still gives a high UR due to the strong current, i.e. $M = 1.02$.

Table 4 Case (1) design wave (height & peak period) and current velocity return period values

	1 year	10 year	100 year
H_s	10.3	13.1	16
T_p	14.2	15.9	17.6
U_r	0.4	0.5	0.7

Table 2 Case (1) - Characteristic values and random variables; OD 10", WT 14.2mm, WD 330m. Mean value are defined as $\mu_L = \chi_{CL}/(1 + 1.5CoV_L)$ for load variables and $\mu_R = \chi_{CR}/(1 - 1.5CoV_R)$ for strength variables.

Design Parameter	Distribution type	Characteristic value	Mean value (μ)	Coefficient of Variation (CoV)
Significant wave height (H_s) [m] (100yrs)	Normal	16.0	13.06	0.15
Spectral peak period (T_p) [s] (100yrs)	Normal	17.6	14.37	0.15
Steady current near seabed (U_r) [m/s] (10yrs)	Normal	0.5	0.41	0.15
Undrained shear strength (S_U) [Pa]	Normal	15300	19742	0.15
Dry unit soil weight (γ_s) [N/m^3]	Normal	17900	21059	0.10
Submerged unit soil weight (γ_{sw}) [N/m^3]	Normal	10000	11765	0.10
Friction coefficient (μ)	Normal	0.2	0.26	0.15

Table 3 Case (2) - characteristic values and random variables; OD 10", WT 25.3mm, WD 135m. Mean value are defined as $\mu_L = \chi_{CL}/(1 + 1.5CoV_L)$ for load variables and $\mu_R = \chi_{CR}/(1 - 1.5CoV_R)$ for strength variables.

Design Parameter	Distribution type	Characteristic value	Mean value (μ)	Coefficient of Variation (CoV)
Significant wave height (H_s) [m] (100yrs)	Normal	15.0	12.24	0.15
Spectral peak period (T_p) [s] (100yrs)	Normal	16.4	13.39	0.15
Steady current near seabed (U_r) [m/s] (10yrs)	Normal	0.69	0.56	0.15
Undrained shear strength (S_U) [Pa]	Normal	15300	19742	0.15
Dry unit soil weight (γ_s) [N/m^3]	Normal	17900	21059	0.10
Submerged unit soil weight (γ_{sw}) [N/m^3]	Normal	10000	11765	0.10
Friction coefficient (μ)	Normal	0.6	0.77	0.15

Table 5 Case (2) design wave (height & peak period) and current velocity return period values

	1 year	10 year	100 year
H_s	10.8	13	15
T_p	14.5	15.5	16.4
U_r	0.6	0.69	0.75

4. Results and discussion
Probabilistic analysis – Case (1)

Case (1) is a pipeline with Outer diameter (OD) = 10.75" (273.1mm) and wall thickness (WT) of 14.2mm in Water Depth (WD) of 330m. The characteristic load is defined as upper fractile, i.e. $\chi_{CL} = \mu_L + 1.5\sigma_L$ while the characteristic resistance as lower fractile, i.e. $\chi_{CR} = \mu_R - 1.5\sigma_R$ for both absolute and generalized lateral stability criteria. Monte Carlo Simulations (MCS) are used for probabilistic analysis. Figure 3 shows the results of MCS with 10^4 simulations with seven random variables as defined in Table 2. The probability of exceeding the characteristic design value is shown in Table 6.

To verify that the number of simulations are sufficient for this type of probabilistic analysis, a different run with 10^5 simulations has been performed. The results show that the percentage of exceeding the characteristic value for the absolute lateral stability criterion is reduced from 1.18% to 1.08%. Hence, the 10^4 simulations seem to be sufficient for the comparison purpose.

The results evidently show that the probability of having exceeded the characteristic design value is marginally small, however this can be influenced by the choice of characteristic design value. Therefore, it is important that especial attention is given to the choice of characteristic value that properly account for individual random variable defined.

The Normal Safety Class for SLS criterion, per DNV-OS-F101 (2013), corresponds to a failure probability level of 1.0×10^{-3}. This means that for the Case (1), when the absolute Stability is concerned, the $UR_{AS,1}$ should be 1.48 or $W_{s,lat,req.}$ should be 984 N/m. If a higher safety class is required, the required submerged weight of the pipeline should even be higher.

Table 6 Probability of exceeding the characteristic design value

	Probability of Exceeding the Characteristic Value	
	Case (1)	Case (2)
$UR_{AS,1}$	1.18×10^{-2}	6×10^{-3}
$UR_{AS,2}$	2.34×10^{-2}	1.42×10^{-2}
$UR_{Gs,Acc.}$	1.69×10^{-2}	3.1×10^{-3}
$W_{s,lat,req.}$	1.18×10^{-2}	5.1×10^{-3}
$W_{s,Acc.,all.}$	1.67×10^{-2}	3.1×10^{-3}
y	1.77×10^{-2}	3.7×10^{-3}

Probabilistic analysis – Case (2)

Case (2) is a pipeline with OD 10" (273.1mm) and wall thickness (WT) of 25.3mm in WD of 135m. The characteristic load is defined as the upper fractile, i.e. $\chi_{CL} = \mu_L + 1.5\sigma_L$ while the characteristic resistance as the lower fractile, i.e. $\chi_{CR} = \mu_R - 1.5\sigma_R$ for both the absolute and the generalized lateral stability criteria.

Figure 4 shows the results of MCS with 10^4 simulations with seven random variables as defined in Table 3. The probability of exceeding the characteristic design value is shown in Table 6. Note here that $UR_{GS,Acc}$ is relevant for Case (2).

As can be seen from Figure 4, there is still a small probability that the design will not be fulfilled due to the randomness involved in the design parameters.

Interestingly, the effect of randomness in the variables is less pronounced for Case (2) than Case (1) due to the increased interaction of waves and currents in shallower waters and sandy soils for case (2), i.e. accounting for the passive soil resistance.

Table 7 shows a summary of the correlation coefficient between URs and design parameters for both cases.

Table 7 Correlation coefficient between UR and design parameters.

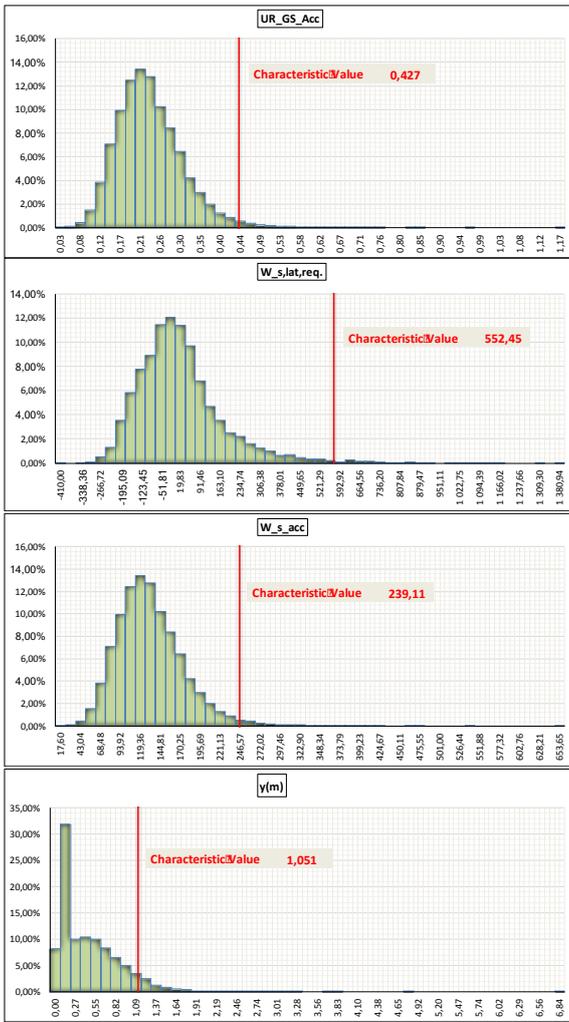|	Case (1)		Case (2)	
	$UR_{AS,1}$	$UR_{Gs,Acc.}$	$UR_{AS,1}$	$UR_{Gs,Acc.}$
H_s	0.18	0.13	0.34	0.3
T_p	0.76	0.44	0.88	0.83
U_r	0.31	0.78	0.1	0.42
S_u	0.03	0.1	0.00	0.00
γ_s	0.00	-0.05	0.00	0.00
γ_{sw}	0.00	0.00	0.00	-0.02
μ	-0.14	0.00	-0.16	0.00
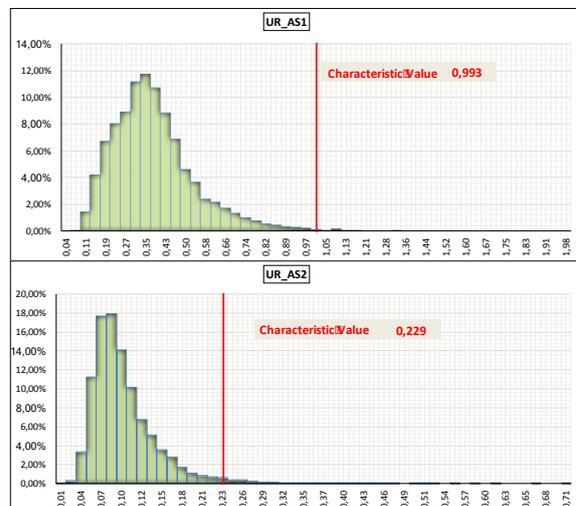
Figure 3 Probability distributions and associated characteristic values for Case (1) OD 10.75", WT 14.2mm, WD 330m - (ref. Table 2)
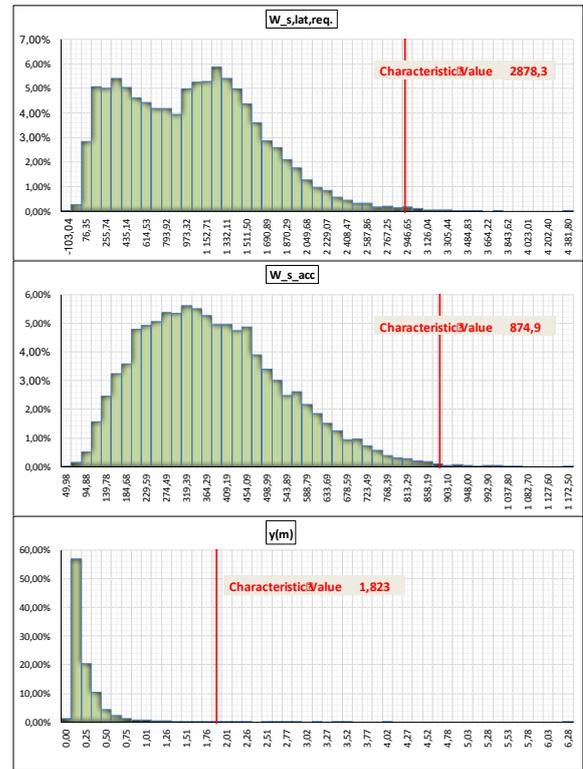

Figure 4 Probability distributions and associated characteristic values for Case (2) OD 10.75", WT 25.3mm, WD 135m (ref. Table 3)

It is observed that:

    − <u>Absolute Stability Criterion:</u>
For Case (1), there is a high correlation between UR and the spectral peak period and medium correlation between UR and the steady current velocity. For Case (2), however, the significant wave height has a higher correlation than the current velocity. This is related to the interaction of waves and currents in shallower water, i.e. Case (2). Friction factor has also some impact on the UR.

    − <u>Generalised Lateral Stability:</u>
For Case (1), the highest correlation is between UR and the current velocity, with the spectral peak period being the next highest. It is noted that, the correlation between UR and the spectral peak period is almost doubled in Case (2) as compared to Case (1). Also, the current velocity becomes more influential than the significant wave height. This is related to the interaction of waves and currents and the sandy soil effect, i.e. the passive soil resistance effect.

It is also noted that, for Case (2), in order to fulfil the required safety level for the SLS criterion, i.e. 1.0×10^{-3} for normal safety class, the $UR_{Gs,Acc.}$ should be 0.86 or $W_{s,Acc.,all.}$ should be 975 N/m. Note here that the generalised lateral stability approach has not been calibrated to reflect the differences in the safety class, i.e. the SC factor is not included in the design format for GS.

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Sensitivity studies
Some sensitivity cases have also been considered here. The sensitivities are defined for Case (2) varying the Diameter of pipeline while the WT kept the same, i.e. 25.3mm. This is done to reflect that the increase in the submerged weight of the pipe is provided through the increase in diameter. The results are shown in Table 8.

It is observed that the results are not very sensitive to the change in diameter and the safety level is consistent in this respect. However, the soil type has an order of magnitude impact on the safety level.

Table 8 Sensitivity cases – Variation is made on the "OD" for Case (2)

		Char. value	Probabilit y of exceeding
OD12"	$UR_{Gs,Acc.}$	0.737	$2.7×10^{-3}$
	y (m)	1.65	$2.7×10^{-3}$
OD14"	$UR_{Gs,Acc.}$	0.701	$3.2×10^{-3}$
	y (m)	1.52	$3.1×10^{-3}$
OD16"	$UR_{Gs,Acc.}$	0.677	$3.4×10^{-3}$
	y (m)	1.48	$3.7×10^{-3}$
OD16" (very loose sand; γ_{sw}=7000 N/m³)	$UR_{Gs,Acc.}$	0.754	$7.0×10^{-4}$
	y (m)	2.12	$7.0×10^{-4}$

5. Conclusions
Uncertainties associated with the on-bottom stability design of submarine pipelines are investigated considering current design practice per DNV-RP-F109 (2011). Important sources of the uncertainties are identified and the sensitivity of the pipeline safety to main design parameters is studied from the stability point of view. Monte Carlo Simulations (MCS) are performed as the basis for probabilistic assessment of the on-bottom lateral stability against the absolute stability and the generalized lateral stability criteria. The assessments are based on simplified assumptions regarding uncertainties in loads and resistance variables, namely, the significant wave height, the peak spectral period, the steady current velocity near seabed, the undrained shear strength, the dry/submerged unit soil weight and the soil friction coefficient.

Uncertainties in the significant wave height and the spectral peak period are found to be important parameters in describing the UR distribution. It is found that the distribution of UR also depends on the soil type, i.e. the passive resistance in the pipe-soil interaction model, which informs what design criterion is more relevant. Therefore, the definition of characteristic values of both loads and resistance variables are important for the UR.

Based on the assumptions made herein, the calculations show that there can be a probability that the design would be un-conservative, i.e. exceeding the characteristic value of the submerged weight calculated with characteristic values being defined as the upper and lower fractile of the distribution for, respectively, load and resistance variables. This is mainly due to the uncertainty in the design variables and the definition of the characteristic values. However, it should be noted that the results are sensitive to the assumptions regarding the characteristic values, and that the actual data can be scarce. There is a high correlation between the spectral peak period and the UR and medium correlation between the steady current velocity and the UR for the generalized lateral stability.

Normal distribution is assumed for all random variables. This is subject to uncertainty and has been assumed here for the sake of benchmarking the effect of randomness on the results. However, some random variables, such as the significant wave height and the soil resistance, have typically skewed distributions rather than normal distribution. In future work, more effort should be made in collecting and systemizing the definition of uncertainties for both loads and resistance variables as well as characteristic variables. In this way, more realistic assessments of the on-bottom lateral stability of the pipelines will be performed and thus appropriate safety formats can be achieved to ensure a safe, economic and reliable pipeline design.

6. Acknowledgment

7. List of Symbols
Acc.	Accumulated
ALS	Accidental Limit State
AS	Absolute Stability
FORM	First Order Reliability Method
GS	Generalised Stability
LRFD	Load and Resistance Factors Design
MCS	Monte Carlo Simulation
NBO	No-Break Out
OD	Outer Diameter
QRA	Quantitative Risk Analysis
SC	Safety Class
SLS	Serviceability Limit State
SORM	Second Order Reliability Method
SRA	Structural Reliability Analysis
UR	Utilisation Ratio
VBA	Visual Basis Application
WT	Wall Thickness
WD	Water Depth

8. Disclaimer
This paper is the result of the analysis carried out by the author as an independent researcher. The paper does not purport to represent the views or the official policy of any organization or company the author have been working for before or currently working.

The information presented in this paper has not been verified against any approved method neither against any test result. Users may utilize the approach at their own risk.

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