Design of Control Strategy for Swarm Autonomous Vessels for Circling Mission in Calm Water

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ABSTRACT

Control of a group of autonomous surface vessels, called agents, with realistic dynamic for circling mission is addressed with the aid of Lyaponov and graph theory. In this brief, to obtain a cooperative controller in between agents, new coordination transfer are presented and graph theory is used to illustrate communication between the agents. With the aid of Lyaponov theory and graph theory application, decentralized and scalable controllers are designed for group of autonomous vessels to converge to a desired geometry for circling around a specific target point. Due to the realistic agent dynamics, non-holonmic dynamics and turning constrains of the vessels are considered in the design process. Advantage of the proposed controller is: it uses domestic information between agents and the controller is designed based on these information. The agents herein represent a large class of autonomous vessels with realistic limitation on vessel motion. Besides, in previous works inertia and damping matrix of the agents were assumed to be diagonal and constant, in this research work non-diagonal inertia matrix and variable damping matrix are under consideration. MATLAB and Simulink are used to represent the effectiveness of the proposed controllers. As the simulation results show, designed controllers perform well on the system and the objective duty is achieved appropriately.

1. Introduction

As autonomy and unmanned systems grow, autonomous vessels have fascinated and attracted the interest of researchers for many years. These systems are recognized as unmanned vessels for many potential and defence application including search and rescue operations, surveillance and others.

A multi agent system is computerized system composed of multiple interacting agents within an environment. Multi agent systems can be used to solve problems that are difficult or impossible for an individual agent or monolithic systems to solve. Most of the researches are focused on studies on the dynamic and control of multi agent systems. Therefore, design of control strategy for this kind of systems is interested in recent decade. A great deal effort has been directed at developing centralized and decentralized control strategy for wide variety of swarm application. In [1] artificial potential and virtual leaders are used for swarm control. Null space based behavioral control are explained in [2] for formation control of under actuated surface vessels. In

[3] back stepping method are addressed and in [4] authors use sliding mode tracking control for surface vessels. Authors in [5] used modified bees algorithm, and in [6] used distance estimation schemes, and in [7] local adaptive internal model based controllers are presented. In this paper, Lyaponov and graph theory are used for design of control strategy for swarm formation of multiple vessels in calm water. This method is based on [8], but cannot be used directly because the model which is selected here, is a real and has more complicated dynamics influences. In other hand, the controller designed in [8] is used for small surface vessels but the model used in this paper is the real tug boat, has a total length of 30 meters.

The remaining of the paper is organized as; in section 2 dynamic model of the tug boat is derived and then coordination transfer is proposed. Mission statement is addressed in section 3 and then cooperative controllers are proposed in section 4. Finally, simulation results are depicted in section 5.

2. Model Dynamic

Consider the *N* actuated three DOF planar vessels which is illustrated in figure 1. Surge propulsive force is delivered by a propeller and a rudder provide torque capable of affecting yaw. Dynamics model of each vessel can be given in equation 1 to equation 7 appropriately [9] and dynamics equations for the *i*'th vessels can be written as:

$$\eta_i = \begin{bmatrix} x_i \\ y_i \\ \theta_i \end{bmatrix} \ and \ v_i = \begin{bmatrix} u_i \\ v_i \\ r_i \end{bmatrix} \quad (1)$$

$$\dot{\eta}_i = R(\theta_i)v_i \quad (2)$$

$$R(\theta_i) = \begin{bmatrix} \cos(\theta_i) & -\sin(\theta_i) & 0 \\ \sin(\theta_i) & \cos(\theta_i) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (3)$$

$$M_i \dot{v}_i + \big(C_i + D_i(v_i)\big)v_i = \begin{bmatrix} \tau_1 \\ 0 \\ \tau_3 \end{bmatrix} \quad (4)$$

$$M_i = \begin{bmatrix} m_{i11} & 0 & 0 \\ 0 & m_{i22} & m_{i23} \\ 0 & m_{i32} & m_{i33} \end{bmatrix} \quad (5)$$

$$D_i = \begin{bmatrix} d_{i11} + d_{i11}^n |u_i| & 0 & 0 \\ 0 & d_{i22} & d_{i23} \\ 0 & d_{i32} & d_{i33} \end{bmatrix} \quad (6)$$

Where:

- (x_i, y_i) is the location of mass center
- L_i is the total vessel length
- L_{cgi} is the distance from the mass center to the vessel bow
- u_i is the surge speed of the vessel in its body frame
- v_i is the sway speed in the body frame
- θ_i is the yaw angel in the world coordinate frame
- τ_1 is the thrust generated by the prime mover
- τ_3 is the yaw torque generated by the rudder system
- M_i is the inertial matrix for vessel *i*
- D_i is the damping matrix for vessel *i*
- C_i is the term representing the Coriolis and centrifugal forces.

The control inputs for this under actuated system are τ_1 and τ_3 which have to control three state variables of the system through a nonlinear equations. These equations also consist nonholonomic constraints which bring limitation on velocities to be performed. Every vessel will be operating in displacement mode, so the weight of vessel is supported by buoyant force.
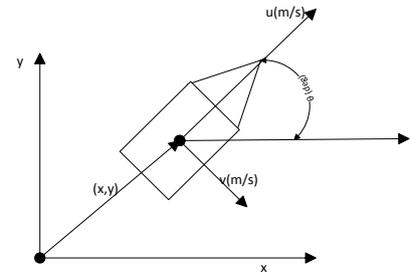

Fig. 1.   Autonomous surface vessel model

3. Mission Statement

Considering *N* under actuated surface vessels. During control mission, each vessel knows its own state and the states of some neighboring vessels by communication links [8]. This communication topologies will be described by the aid of graph theory. It is assumed that all agents know its owns states and the states of some other vessels.

Given a desired geometric pattern P defined by constant vectors $[p_{jx}, p_{jy}]$ for $1 \le j \le n$, assuming:

$$\sum_{j=1}^{n} p_{jx} = 0, \qquad \sum_{j=1}^{n} p_{jy} = 0 \quad (8)$$

Based on the desired trajectory, designing a cooperative controller for each vessel such that [8]:

$$\lim_{t \to \infty}\left(\begin{bmatrix} x_i - x_j \\ y_i - y_j \end{bmatrix} - \begin{bmatrix} p_{ix} - p_{jx} \\ p_{iy} - p_{jy} \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (9)$$

$$\lim_{t \to \infty}\big(\theta_j - \theta_d\big) = 0 \quad (10)$$

$$\lim_{t \to \infty}\big(u_j - u_d\big) = 0 \quad (11)$$

$$\lim_{t \to \infty}\big(v_j - v_d\big) = 0 \quad (12)$$

$$\lim_{t \to \infty}\big(\dot{\theta}_i - \dot{\theta}_d\big) = 0 \quad (13)$$

To achieve these control targets, coordination transfer is proposed here. Following the work [8], the new errors are defined base on system dynamics. The new errors are defined as:

$$\lim_{t \to \infty} e_j = \lim_{t \to \infty}\left(\begin{bmatrix} x_j \\ y_j \\ \theta_j \\ u_j \\ v_j \\ \dot{\theta}_j \end{bmatrix} - \begin{bmatrix} x_d + p_{jx} \\ y_d + p_{jy} \\ \theta_d \\ u_d \\ v_d \\ \dot{\theta}_d \end{bmatrix}\right) = 0 \quad (14)$$

So the cooperative controller in between agents

should be design in such a way that the above control scheme to be achieved. Based on the agents' dynamic, new tracking errors are defined and the controller will be tuned based on these new errors. Tracking errors are defined as:

$$\delta_{1j} = (x_j - p_{jx})\cos(\theta_j)$$
$$+ (y_j - p_{jy})\sin(\theta_j)$$
$$- (x_d \cos(\theta_d) \tag{15}$$
$$+ y_d \sin(\theta_d)$$

$$\delta_{2j} = v_j - v_d \tag{16}$$

$$\delta_{3j} = -(x_j - p_{jx})\sin(\theta_j)$$
$$+ (y_j - p_{jy})\cos(\theta_j)$$
$$+ \frac{m_2}{d_2}v_j - (-x_d \sin(\theta_d) \tag{17}$$
$$+ y_d \cos(\theta_d) + \frac{m_2}{d_2}v_d)$$

$$\delta_{4j} = \theta_j - \theta_d \tag{18}$$

$$\delta_{5j} = \dot{\theta}_j - \dot{\theta}_d + k_4\delta_{4j} \tag{19}$$

$$\delta_{6j} = -\frac{m_1}{d_2}u_j - \delta_{1j} + \frac{m_1}{d_2}u_d + k_3\delta_{5d}\delta_{3j} \tag{20}$$

These new inputs coordinates are different from those three in planar maneuver of any vessel [8]. These transfer coordinates are defined as:

$$w_{1j} = \left(\frac{d_1}{d_2} - 1\right)u_j - \delta_{3j}\delta_{6j} - \frac{\tau_{1j}}{d_2}$$
$$- (-m_{32}\dot{v}_d - \frac{d_{32} + c_{32}}{m_{32}}v_d \tag{21}$$
$$- \frac{c_{33}}{m_{33}}\dot{\theta}_d + \frac{\tau_{2d}}{m_{33}})$$

$$w_{2j} = -m_{32}\dot{v}_j - \frac{d_{32} + c_{32}}{m_{32}}v_j - \frac{c_{33}}{m_{33}}\dot{\theta}_j$$
$$+ \frac{\tau_{2j}}{m_{33}} - (\left(\frac{d_1}{d_2} - 1\right)u_d \tag{22}$$
$$- \delta_{3d}\delta_{5d} - \frac{\tau_{1d}}{d_2})$$

The new errors are defined as $\delta_* = [\delta_{1j}, \delta_{2j}, \delta_{3j}, \delta_{4j}, \delta_{5j}, \delta_{6j}]$. It is easy to show that if $\lim\limits_{t\to\infty} \delta_* = 0$, then $\lim\limits_{t\to\infty} e = 0$ achived. The dynamics of tracking errors are:

$$\dot{\delta}_{1j} = -\frac{d_2}{m_1}\delta_{1j} - \frac{d_2}{m_1}(\delta_{6j} - k_3\delta_{5j}\delta_{3j})$$
$$+ \delta_{3j}(\delta_{5j} - k_4\delta_{4j})$$
$$+ \delta_{3d}(\delta_{5j} - k_4\delta_{4j})$$
$$+ \delta_{5d}\delta_{3j} \tag{23}$$
$$- \frac{m_2}{d_2}[\delta_{2j}(\delta_{5j} - k_4\delta_{4j})$$
$$+ \delta_{2d}(\delta_{5j} - k_4\delta_{4j})$$
$$+ \delta_{2j}\delta_{5d}]$$

$$\dot{\delta}_{2j} = -\frac{d_2}{m_2}\delta_{2j} + \frac{d_2}{m_2}[\delta_{1j}(\delta_{5j} - k_4\delta_{4j})$$
$$+ \delta_{1d}(\delta_{5j} - k_4\delta_{4j})$$
$$+ \delta_{1j}\delta_{5d}$$
$$+ (\delta_{5j} - k_4\delta_{4j})(\delta_{6j} \tag{24}$$
$$- k_3\delta_{5d}\delta_{3j})$$
$$+ (\delta_{5j} - k_4\delta_{4j})\delta_{6d}$$
$$+ \delta_{5d}(\delta_{6j} - k_3\delta_{5d}\delta_{3j})]$$

$$\dot{\delta}_{3j} = (\delta_{5j} - k_4\delta_{4j})(\delta_{6j} - k_3\delta_{5d}\delta_{3j})$$
$$+ (\delta_{5j} - k_4\delta_{4j})\delta_{6d} \tag{25}$$
$$+ \delta_{5d}(\delta_{6j} - k_3\delta_{5d}\delta_{3j})$$

$$\dot{\delta}_{4j} = -k_4\delta_{4j} + \delta_{5j} \tag{26}$$

$$\dot{\delta}_{5j} = w_{1j} - w_{1d} + k_4(-k_4\delta_{4j} + \delta_{5j}) \tag{27}$$

$$\dot{\delta}_{6j} = w_{2j} - w_{2d} + k_3\dot{\delta}_{5d}\delta_{3j}$$
$$+ k_3\delta_{5d}(\delta_{5j} - k_4\delta_{4j})(\delta_{6j}$$
$$- k_3\delta_{5d}\delta_{3j}) \tag{28}$$
$$+ k_3\delta_{5d}(\delta_{5j} - k_4\delta_{4j})\delta_{6d}$$
$$+ k_3\delta_{5d}^2(\delta_{6j} - k_3\delta_{5d}\delta_{3j})$$

Based on the [8], following lemmas are addressed:
Lemma 1: For the variables defined in the above formulas, if:

$$\lim_{t\to\infty}(\delta_{ij} - c_i) = 0 \quad (1 \le i \le 6, \ 1 \le j \tag{29}$$
$$\le m)$$

Where c_i are bounded variables, then equation 9 is being satisfied. Furthermore, if $c_i = 0$ for$(1 \le i \le 6)$, then equations 9 to 13 are being satisfied.
Lemma 2: For system equations 15 to 20, if δ_{5j} and δ_{6j} exponentially converge to constants c_5 and c_6, respectively, for $1 \le j \le m$, then equation 9 is being satisfied. Furthermore, if $c_5 = c_6 = 0$, then equations 9 to 13 are being satisfied, which means that the cooperative control problem is achieved.

4. Communication Digraph
Assumption 1: The communication digraph G is fixed and has a spanning tree.
Given any $m * m$ constant matrix $A = [a_{ji}]$ with $a_{ji} > 0$ for $1 \le i, j \le m$, the Laplacian matrix $L = [L_{ji}]$ of

the digraph G with weight matrix A is defined as:

$$
L_{ji} = \begin{cases} -a_{ji} & if\ i \neq j\ and\ i \in N_j \\ 0 & if\ i \neq j\ and\ i \in N_j \\ \sum_{l \neq j, l \in N_j} a_{jl} & if\ i = j \end{cases} \tag{30}
$$

Lemma 3: L is the Laplacian matrix of the digraph G with weight matrix $A = [a_{ji}]$ and $a_{ji} > 0$. If the digraph G satisfies Assumption 1, then:

$$
\begin{aligned}
\lim_{t \to \infty} e^{\mu t}\left(e^{-Lt} - 1 w_l^T\right) \\
= 0 \lim_{t \to \infty} e^{\mu t}\left(e^{-Lt} - 1 w_l^T\right) \\
= 0
\end{aligned} \tag{31}
$$

For any $\mu \in \left[0, Re\left(\lambda_2(L)\right)\right]$, where w_l satisfies $w_l^T L = 0$ and $w_l^T 1 = 1$. Accordingly [10].

5. Cooperative Control laws

For the systems in equations 1 to 7, regarding to Lemma 1, 2 and 3 and assumption 1, control laws are:

$$
\begin{aligned}
\tau_{1j} = \sum_i d_2 a_{ji}(\delta_{5j} - \delta_{5i}) - d_2 w_{1d} \\
+ d_2 k_4(-k_4 \delta_{4j} + \delta_{5j}) \\
+ (d_1 - d_2)u_j - d_2 \delta_{3j}\delta_{6j} \\
- d_2(-m_{32}\dot{v}_d \\
- \frac{d_{32} + c_{32}}{m_{32}} v_d - \frac{c_{33}}{m_{33}}\dot{\theta}_d \\
+ \frac{\tau_{2d}}{m_{33}})
\end{aligned} \tag{32}
$$

$$
\begin{aligned}
\tau_{2j} = -\sum_i m_3 a_{ji}(\delta_{6j} - \delta_{6i}) + m_3 w_{2d} \\
- m_3 k_3 \dot{\delta}_{5d}\delta_{3j} \\
- m_3 k_3 \delta_{5d}(\delta_{5j} \\
- k_4 \delta_{4j})(\delta_{6j} - k_3 \delta_{5d}\delta_{3j}) \\
- m_3 k_3 \delta_{5d}(\delta_{5j} \\
- k_4 \delta_{4j})\delta_{6d} \\
- m_3 k_3 \delta^2_{5d}(\delta_{6j} \\
- k_3 \delta_{5d}\delta_{3j}) \\
- (m_1 - m_2)u_j v_j + d_3 \dot{\theta}_j \\
+ m_3\left(\left(\frac{d_1}{d_2} - 1\right)u_d\right. \\
\left. - \delta_{3d}\delta_{5d} - \frac{\tau_{1d}}{d_2}\right)
\end{aligned} \tag{33}
$$

Which $a_{ji} > 0, k_3 > 0, and\ k_4 > 0$.

Group of surface vessels approach to the desired pattern by the control laws in equation 32 and equation 33. Besides, these control laws have decentralized and scalable properties while performing proceed. Simulation results in [8] are used for small crafts and vessels with very simple dynamics, while proposed approach here have more generality.

6. Simulation results

The performance of the control laws are shown in following figures, tables, and discussions. In this research work, 3 surface tug boats which basic dynamic parameters were adopted from [9], are under control to converge to a desired pattern in distance from the target. Assume that the communication digraph is fixed. Control parameters are as: $a_{ij} = 1, k_3 = 20\ and\ k_4 = 20$. Initial conditions corresponding to the vessels are given in table I.

TABLE I. . INITIAL CONDITION OF 3 SURFACE VESSELS

$[x_1(0), x_2(0), x_3(0)]$	[25,25,25][m]
$[y_1(0), y_2(0), y_3(0)]$	[20,30,25][m]
$[\theta_1(0), \theta_2(0), \theta_3(0)]$	$\left[\frac{\pi}{4}, \frac{\pi}{6}, 0\right]$[rad]
$[u_1(0), u_2(0), u_3(0)]$	[0,0,0][m/s]
$[v_1(0), v_2(0), v_3(0)]$	[0,0,0][m/s]
$[r_1(0), r_2(0), r_3(0)]$	[0,0,0][rad/s]

Figure 2 demonstrates the entire formation trajectories of the systems. Figure 3 and 4 show that the control efforts with surge force and steering torque are bounded. Consequently, figures 3 and 4 confirms that the level of control activity is reasonable, and no saturation has occurred during the process. Figure 5 show that the velocities of the every agents during the circling mission are in reasonable margins and do not exceed to infinite values. Figures 6 shows excellent error convergences after an initial transient error associated with the nonholonomic nature of the unit agents. Figure 7 and 8 show the trajectory path of the fore and seven agents. They demonstrate the scalability of the system.
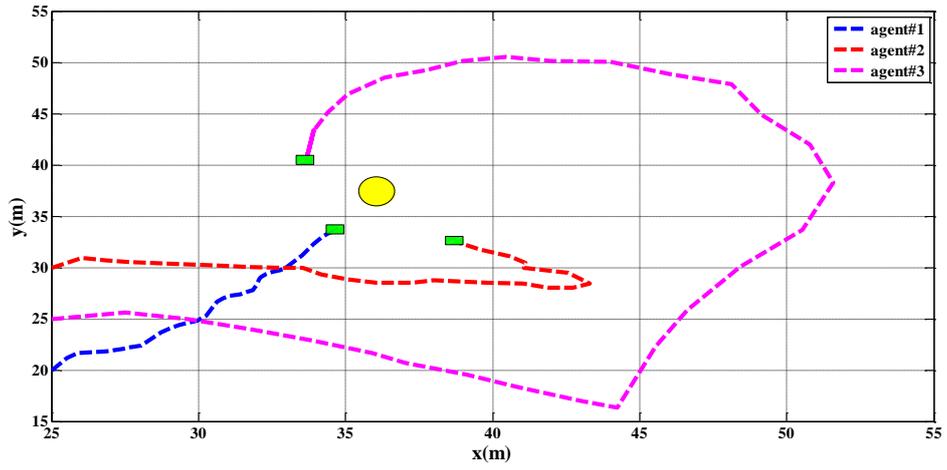
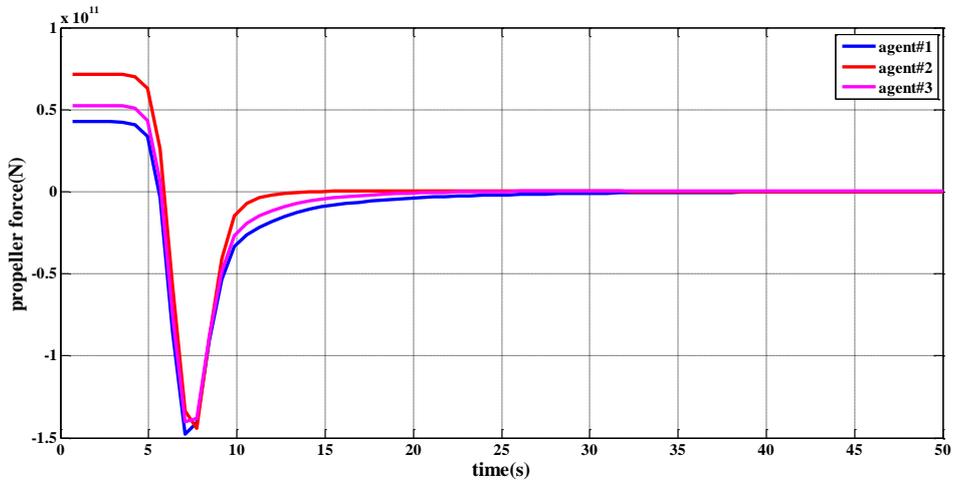**Fig. 2.** Formation trajectory of the system circling around target.

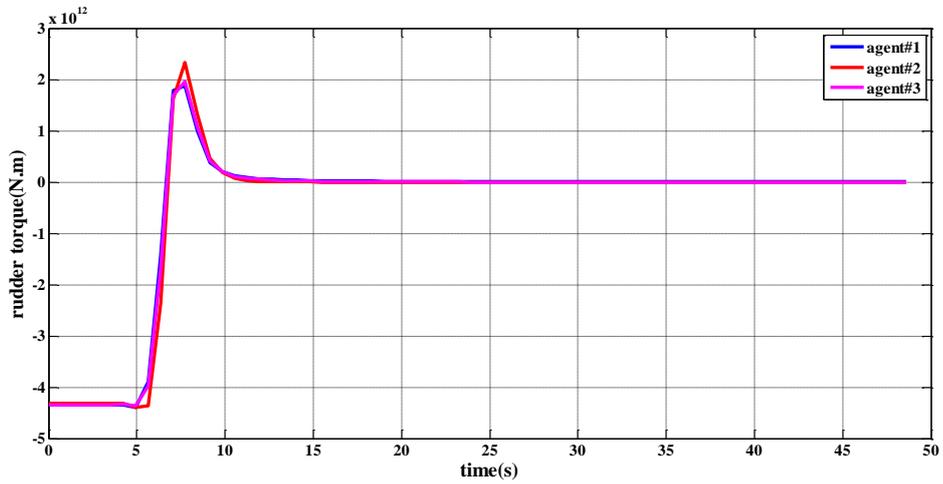**Fig. 3.** Surge force time history.

Fig. 4. . Steering torque time history.
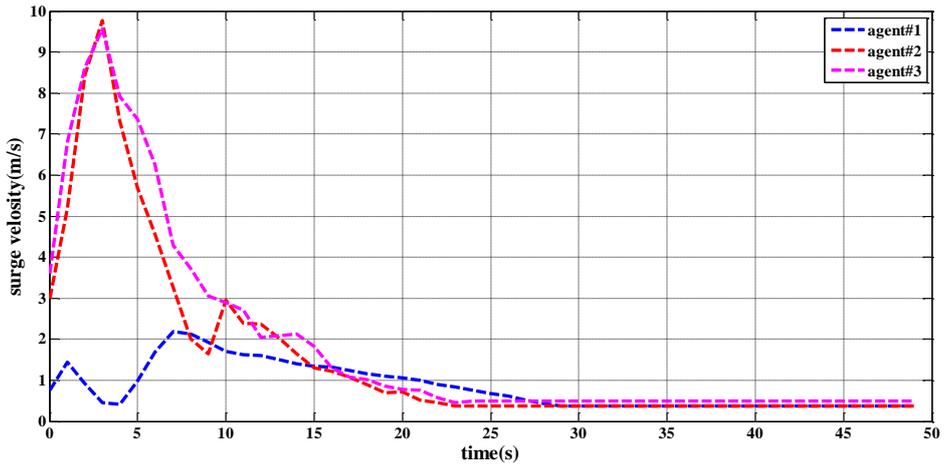
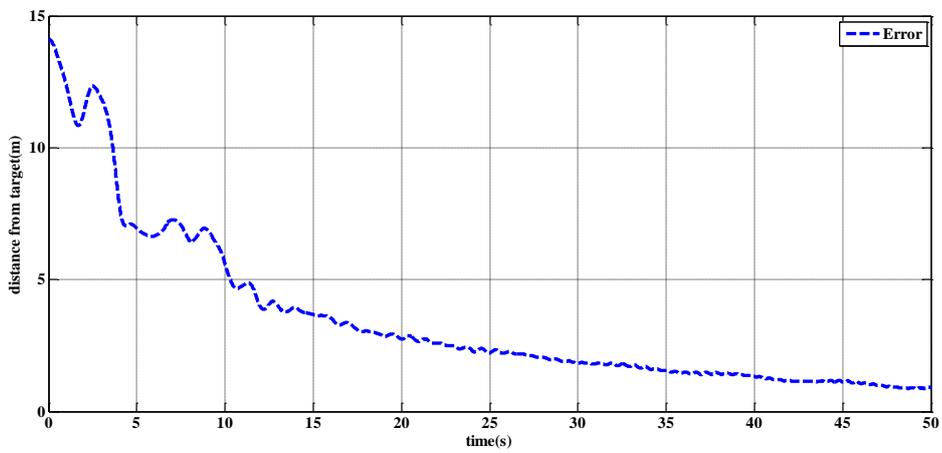**Fig. 5.   Surge velocity .**

Fig. 6.   The distance of the swarm center from target position
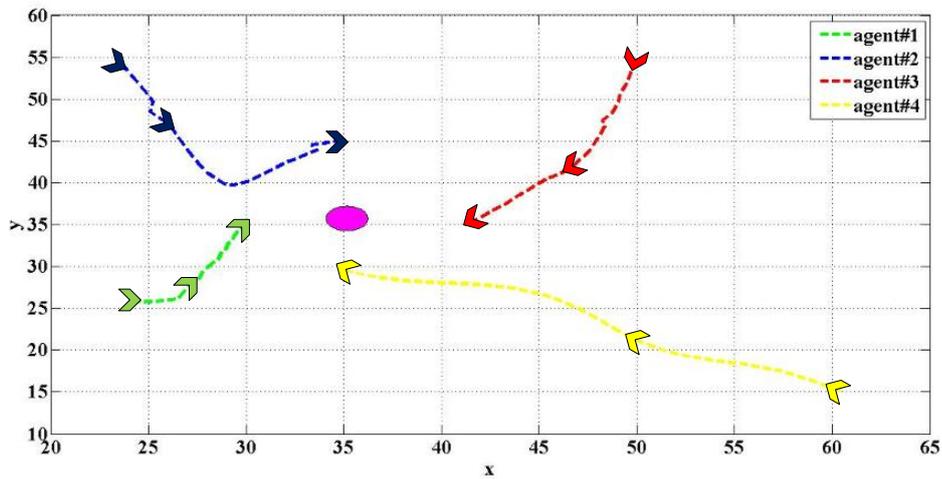

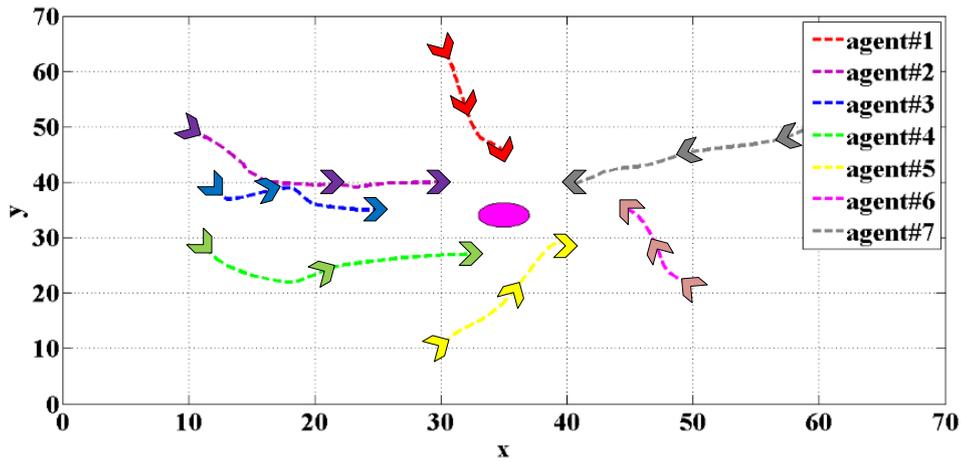**Fig. 7.   Trajectory and coordinated control of four agents**

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Fig. 8.   Trajectory and coordinated control of seven agents

7. Conclusion

In this paper, a decentralized and scalable cooperative controller is proposed for a group of surface vessels with Non-Holonomic dynamic to converge to desired pattern in calm water. The controller laws are based on Lyaponov and Graph theorems, using suitable coordinate transformation. As the simulation results show, effectiveness of the proposed controller laws is achieved. Due to high order of Hydrodynamic matrix elements of the agents, level of control activities are reasonable and no saturation occurs during the mission.

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