Modeling and Control of Autonomous Underwater Vehicle (AUV) In Heading and Depth Attitude via PPD Controller with State Feedback

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1. Introduction

Autonomous underwater vehicle (AUV) is an unmanned submersible in different sizes. It is intended to provide scientists and researchers with simple, low-cost, medium and long-range, appropriate time response capability to collect environmental data. There are a lot of applications for AUVs, including oil industry, survey on underwater animals and plants, operations in dangerous waters, photometric survey, pipeline route survey, seabed mapping, environmental monitoring, chemical plume tracing [1], salvage and rescue requests, and so on [2]. Today the significance of AUVs can easily be understood if the Unmanned Underwater Vehicles (UUV) program for the US Navy is studied [3]. Derivation of AUV parameters is a difficult process and finally its validation demands to analyze practical test like two tank test [4] or telemetry under sea [5, 6]. However, at last, estimation of parameters has uncertainty and variation; therefore, the controller must be self-tuning and robust in counter to variation of AUV parameters and also unpredictable environmental disturbances. Inherently, nonlinear dynamics of AUVs make it more difficult to exert commonly used linear control. The dynamic characteristics of an AUV are quite complex due to its high nonlinearity, time-varying dynamic behavior, uncertainties in hydrodynamic coefficients, and disturbances caused by sea currents and waves. Throughout the years various models of control techniques have been proposed. This includes linear controllers [1, 2, 7, 8, 10, 23], which have performed satisfactorily; SMC controllers [11, 13], adaptive control [12, 13, 20], FLC (Fuzzy Logic Control) [14], predictive control [18–19], static feedback control [25], and neural-network-based control [15–17] have also shown good robustness and tuning ability. Since almost all control methods have some pros and cons, a proper controller can be achieved by the combination of classical and modern intelligent method. One of the most important disadvantages of linear controllers like LQR and LQG is that they are unable to account for the nonlinearities of the system, thus they can result in suitable performance and even instability in high maneuver treatments. Also, neural network has some weak points that bind its improvement. It converges to a precise model with long training time and slow rate, which is not acceptable by many systems. Also, classical neural network does not qualify the main requirements such as fast response, less overshoot–undershoot.

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SMC is an earlier method that is a good solution for nonlinear system but it can cause chattering on actuators, waste energy, and make fault on fins. However, there are some methods like combination with fuzzy or changing the sign functions by saturation function to reduce chattering.

The FLC is easy to use in industrial process because of its simple control structure, easy and cost-effective design [24]. However, FLC with fixed scaling factors and fuzzy rules may not give complete performance index than classical controllers; however, they may require neat to exact models. The main aim of this paper is to develop an attitude control system of an AUV based on Myring hull profile via using PPD controller with state feedback. The goals are to:

1. Understand the general dynamic and kinematic of AUV by using MATLAB 2014
2. Achieve a suitable and simple controller to control heading and depth
3. Simulate the AUV’s maneuvers (Snake and Spiral) and analyze the performance and accuracy of controller.

The advantage of using a PID controller is it is simple to implement and maintain, however, it is primarily applicable for linear time-invariant systems, though many extensions to nonlinear systems have been made such as [21] and references therein.

2. Model description

2.1. Coordinate systems and kinematic and dynamic equations of motion

Generally, the motion of an AUV can be introduced by six degrees of freedom (6-DOF) differential equations of motion [4], [22]. These equations are developed using two coordinate frames shown in Fig. 1. Six velocity components [u, v, w, p, q, r] (surge, sway, heave velocity, roll, pitch rate, yaw rate) are defined in the body fixed frame, while the earth-fixed frame defines the corresponding attitudes and positions [x, y, z, φ, θ, and ψ]. It is listed in Table 1. The axis is right-handed. The origin of the body-fixed coordinate system is center of mass.

![Fig. 1 Reference frame of AUV](image-url)

Table 1 Symbols used to describe 6-DOF

<table>
<thead>
<tr>
<th>DOF</th>
<th>Motion</th>
<th>Forces and moments</th>
<th>velocity</th>
<th>Positions and Euler angles</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>surge</td>
<td>X</td>
<td>u</td>
<td>x</td>
</tr>
<tr>
<td>2</td>
<td>sway</td>
<td>Y</td>
<td>v</td>
<td>y</td>
</tr>
<tr>
<td>3</td>
<td>Heave</td>
<td>Z</td>
<td>w</td>
<td>z</td>
</tr>
<tr>
<td>4</td>
<td>roll</td>
<td>K</td>
<td>p</td>
<td>φ</td>
</tr>
<tr>
<td>5</td>
<td>Pitch</td>
<td>M</td>
<td>q</td>
<td>θ</td>
</tr>
<tr>
<td>6</td>
<td>Yaw</td>
<td>N</td>
<td>r</td>
<td>ψ</td>
</tr>
</tbody>
</table>

The AUV motion is described by these vectors:

\[
\eta = \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix}, \quad \eta_1 = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ position vector}, \quad \eta_2 = \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix} \text{ Euler angles vector} \\
\nu = \begin{bmatrix} \nu_1 \\ \nu_2 \end{bmatrix}, \quad \nu_1 = \begin{bmatrix} u \\ v \\ w \end{bmatrix} \text{ linear velocity vector}, \quad \nu_2 = \begin{bmatrix} p \\ q \\ r \end{bmatrix} \text{ angular velocity vector} \\
\tau = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}, \quad \tau_1 = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \text{ forces vector}, \quad \tau_2 = \begin{bmatrix} K \\ M \\ N \end{bmatrix} \text{ moments vector}
\]

It should be considered that in order to avoid singularity in circulation and transformation, Euler angles should be in this boundary:

\[
0 \leq \psi < 2\pi \\
-\frac{\pi}{2} < \theta < \frac{\pi}{2} \\
-\pi < \phi \leq \pi
\]

Transformation between these two coordinate systems is as follows:

\[
\begin{align*}
\eta &= \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} \\
\eta_1 &= \begin{bmatrix} x \\ y \\ z \end{bmatrix} \\
\eta_2 &= \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix} \\
\nu &= \begin{bmatrix} \nu_1 \\ \nu_2 \end{bmatrix} \\
\nu_1 &= \begin{bmatrix} u \\ v \\ w \end{bmatrix} \\
\nu_2 &= \begin{bmatrix} p \\ q \\ r \end{bmatrix} \\
\tau &= \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} \\
\tau_1 &= \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \\
\tau_2 &= \begin{bmatrix} K \\ M \\ N \end{bmatrix}
\end{align*}
\]
\[ J_1(\eta_2) = C_{\gamma,\psi}^T \ast C_{\gamma,\phi}^T \ast C_{\gamma,\theta}^T = \\
\begin{bmatrix}
\cos(\psi) & -\sin(\psi) & 0 \\
\sin(\psi) & \cos(\psi) & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\cos(\theta) & 0 & \sin(\theta) \\
0 & 1 & 0 \\
-\sin(\theta) & 0 & \cos(\theta)
\end{bmatrix}
\]
\[ x = u \left( \cos(\theta) \cos(\psi) \right) + \\
v \left( \sin(\phi) \sin(\theta) \cos(\psi) - \cos(\phi) \sin(\psi) \right) + \\
w \left( \cos(\phi) \sin(\theta) \cos(\psi) + \sin(\phi) \sin(\psi) \right)
\]
\[ \dot{y} = u \left( \cos(\theta) \sin(\psi) \right) + \\
v \left( \sin(\phi) \sin(\theta) \sin(\psi) + \cos(\phi) \cos(\psi) \right) + \\
w \left( \cos(\phi) \sin(\theta) \sin(\psi) - \sin(\phi) \cos(\psi) \right)
\]
\[ \dot{z} = -z' = u (\sin(\psi)) + \\
v \left( -\sin(\phi) \cos(\theta) \right) + \\
w \left( -\cos(\phi) \cos(\theta) \right)
\]

3. Controller Design:

Based on the linear model containing 3 states, we will design a proportional - derivative (PD) inner loop for controlling the pitch angle \( \theta \), and a proportional (P) outer loop for controlling the depth \( z \).

3.1. Vessel Transfer Function

The first step to design control system for a vehicle is obtain its transfer function. At first, we want to achieve the inner loop transfer function that relating input stern angle \( f_s \) to the output vehicle pitch angle \( \theta \). According to kinematic and dynamic equations expressed, the inner Pitch loop transfer function is as follows:

\[ G_{\theta}(s) = \frac{\theta(s)}{f_s(s)} = \frac{\frac{M_f}{I_y} - \frac{M_q}{I_y}}{s^2 - \frac{I_y}{M_q}s - \frac{I_y}{M_q}} \]

Now we calculate outer depth loop transfer function that relating the input Pitch angle \( \theta_d \) to vehicle depth \( z \). Under real conditions, the inner Pitch loop response is much faster than the outer depth loop so we can make equal with \( \Theta \). According to kinematic and dynamic equations expressed, the depth transfer function is as follows:

\[ G_{\theta}(s) = \frac{Z(s)}{\theta(s)} = \frac{-\mu_1}{s} \]

3.2. Control Law

As denoted before, we want design a proportional-derivative (PD) inner loop for controlling the pitch angle \( \Theta \) and a proportional (P) for controlling the depth \( z \). According to the control law presented above, the inner loop control law is as follow:

\[ f_s(s) = -K_p(T_d s + 1) \]

Where

\[ e_\theta(error in pitch) = \theta_{des} (desired pitch) - \theta(actual pitch) \]

Kp is proportional gain and Td is derivative time constant. Because of difference between the vehicle stern angle and pitch angle the minus sign is applied. Positive stern angle create negative torque around Y
axis that force the vehicle diving down (negative pitch rate).
The control law for outer loop denoted as follow:
\[
\Delta = \frac{\theta(s)}{e_2(s)}
\]  \hspace{1cm} (20)
Where
\[
e_z = z_d - z
\]  \hspace{1cm} (21)
In view of the above control equations and considering the dynamic and kinematic characteristics of our AUV model, we designed the vehicle P-PD controller system blocks that shown as below:

![Figure 2: The vehicle controller system block](image)

3.3. Controller design procedure:
Before choosing gain of the controller for a specified vessel we review the controller design. We design our own controller to have a specified second order answer in natural frequency \(\omega_n\) and damping coefficient \(\zeta\). For second order answer we have:
\[
G(s) = \frac{K}{s^2 + 2\zeta\omega_n s + \omega_n^2}
\]  \hspace{1cm} (22)
3.4. Overshoot, ts, tp
For choosing a suitable answer in a specified system, we must consider the %OS, setting time (ts) and peak time (tp).
Damping coefficient is a function of the percentage overshoot that calculated as follows:
\[
\zeta = \frac{-\ln(\frac{\%OS}{100})}{\pi + \ln\left(\frac{\%OS}{100}\right)}
\]  \hspace{1cm} (23)
By damping coefficient and using the following equations the natural frequency can be obtained:
\[
\omega = \frac{\pi}{tp \sqrt{1-\zeta^2}}
\]  \hspace{1cm} (24)
\[
\omega = \frac{4}{\zeta ts}
\]  \hspace{1cm} (25)
3.5. Obtain the poles:
By considering the transfer function of second order answer and using the following the equation we can determine the location of poles in the coordinate plane:
\[
S_{1,2} = \xi \omega \pm \omega \sqrt{1-\xi^2}
\]  \hspace{1cm} (26)
3.6. Pitch Controller
According to the vessel specifications:
Fin Lift= Mfs = -1575.9 kg.m/s
Iy=469 kg/m2
Added Mass =Mq = -458 kg.m2
Combined Term = Mq = 9826.2 kg.m2/s
Hydrostatic = Mθ = 13719.6 kg.m2/s2
And using \(G_\theta(s)\), the Open loop transfer function is as follows:
\[
G_\theta(s) = \frac{-1.7}{s^2 + 10.68s + 14.85}
\]  \hspace{1cm} (27)
And the open loop poles are:
\(S_1 = -1.654\)
\(S_2 = -8.945\)
Given that the vessels depth changes should not be too much the overshoot chosen around 0.05. By using \(G_\theta(s)\), the value of \(\zeta = 0.69\) and the value of \(T_\theta = 0.21\) was obtained. By using the transfer function Root-Locus graph that we draw in MATLAB, the natural frequency \(\omega_n = 1.27\) rad/sec and \(K_p = 0.2604\) obtained.

![Figure 3: Transfer Function’s Root-Locus Graph](image)

3.7. Depth Controller
The depth transfer function adds third pole to the system. To ensure that the Pitch loop response is faster than depth loop response, the Pitch loop poles must be five-time interval away from the depth loop pole. For this we use a graph of the root-Lucas has achieved the appropriate \(\Delta\).

3.7.1. State feedback controller
In this section we use state feedback controller with trigger signal, \(u = -Kx\). Where \(K\) is gain matrix and \(x\) is state matrix.
We have:
\[
G_\theta(s) = \frac{-1.7}{s^2 + 10.68s + 14.85}
\]  \hspace{1cm} (28)
And
\[
G_x = \frac{-20}{s}
\]  \hspace{1cm} (29)
That gives us:
\[
G_\theta(s) \times G_x = \frac{34}{s^4 + 10.68s^2 + 14.85}
\]  \hspace{1cm} (30)
\([A, B, C, D] = tf2ss ([34],[1 10.6 14.8 0])\)  \hspace{1cm} (31)
gives:
\[
A = \begin{bmatrix}
-10.6 & -14.8 & 0 \\
1 & 0 & 0 \\
0 & 1 & 0
\end{bmatrix}
\]
\[B = \begin{bmatrix}1\end{bmatrix}\]
\[C = [0 \ 0 \ 34]\]
\[D = 0\]
\hspace{1cm} (32)
The controllability matrix \( P_C = [B \ AB \ A^2B] \) is calculated. If the \( P_c \) be a non-zero matrix, which means the system is state controllable.

For \( \zeta = 0.69 \) the poles are \((-0.338 \pm j \ 0.355 \) and 
\(-10 \)
gives us \( s^3 + 10.676s^2 + 7s + 2.4 = 0 \).

When the matrix A is not in CCF form, we use \( \bar{x} = Px \) to transfer the system to CCF form. Then the matrix \( P \) is \([0, 0, 1; 0, 1, 0; 1, 0, 0]\) and the effective system is \((-10.6, -14.8, 0)\). So the gain matrix is as following:

\[
K = [10.676 + 1.09,.7 + 0.52,2.4 - 0 ]
\]

and

\[
P = [11.766, 7.52, 2.4]
\]

### 4. Simulation results

In order to demonstrate the effectiveness and stability of P-PD controller designed in this paper, we tested the maneuverability of selected AUV that we described the specification in section 3.6. One of the most important maneuvers that the underwater vessels do is circulation maneuver that is done by rudders level control. The maneuver is performed by applying a constant angle to the rudder, the submarine began to rotate and its height start decreases. Another important maneuver for submarines is dive and climb maneuvers. These maneuvers are done by using the elevator control levels. These maneuvers change the AUV’s depth.

Two of the most important control levels in submarines are rudder control levels (dr) and elevator control levels (ds). The elevator control levels are control the depth means by applying positive and negative input the vessel will climb and dive. As shown in figures 4, 5 and 6 by using positive ds as controller input after 100 seconds the submarine depth changes from 0 to 43 meters. By applying negative ds in designed controller, the submarine start diving gently (figures 7, 8 and 9). By changing the rudder control elevators we can control the submarines heading. In the figures 10 and 11 by applying dr= 0.1 the submarines do the snake maneuver without any change in depth. The figure 12 shows the Snake maneuver in 3-D plot. By applying both rudder control levels and elevator control levels the submarines do the Circulate maneuver that shows in figures 13 and 14.

#### 4.1. Simulation results for \( d_r = 0.1 \) (rad)

As mentioned before, for positive elevator control levels input the submarine began climb. The simulation results show below:

#### 4.2. Simulation results for \( d_s = -0.1 \) (rad)

For negative elevator control levels input the submarine began dive. The simulation results show below. As is evident from the results for asymmetric control orders, the AUV shown symmetric behavior, which represents the accuracy of the simulation.
4.3. Simulation results for $dr=0.1$ (rad)
For the rudder control input, submarines began to rotate that the results are shown below:
5. Conclusions
Control systems for depth and heading control of a small AUV were designed and tested in field. The P-PD with state feedback has proved useful in the development of the control systems for the AUV based on Myring hull profile. To construct the control system, a 6 DOF general mathematical model of underwater vehicles was derived, which is powerful enough to apply it to different kinds of underwater vehicles according to its own physical properties. Based on the general mathematical model, a simulation platform was established to test motion characteristics, stability and controllability of the vehicle. The control system was design based on AUV’s mathematical model. After that we test the control system performance by dive, climb and circulation orders. The results show that the performance of controllers meets the requirement of performance and stability in both the vertical and horizontal planes.

6. References
[25] Subudhi B, Mukherjee K, Ghosh S, A static output feedback control design for path following of
autonomous underwater vehicle in vertical plane.
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