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Equivalent Half Pulse (EHP) Method for Vibration Analysis under Regular Wave

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ABSTRACT

Fully dynamic analysis of offshore structures under random wave loads in time domain is sometimes necessary for calculating structural responses in design issues. Such analyses are very time consuming and therefor simplified methods for estimation of acceptable response of these structures can be very useful in initial design. In this paper an innovative method to obtain response spectrum of fixed offshore structures caused by extreme waves is represent based on concept of impulse response spectrum. For this purpose the structural system is considered as a simple one degree of freedom structure and the different sea states are equalized to different half sinusoidal pulses. Response spectrum of structure is defined as a plot of structural response to these pulses for different periods. By using this method the cost of computations is decreased significantly while the accuracy of results was preserved.

1. Introduction

Design and assessment of complex structural systems such as fixed offshore structures under environmental loads require heavy time consuming computations. It is desirable for engineers to reduce these cost of analysis while maintain maximum precision in order to obtain reliable results. Such approaches result in creation of procedures like response spectrums for elastic and inelastic design of structures and capacity spectrum method for assessment of structures subjected to seismic loads.

The mathematical formulation of the RSM first appeared in the doctoral dissertation of M.A. Biot in 1932 and in two of his papers .His pioneering work in the 1930's on the response of structures to transient disturbances led to the key concept of response spectrum as a universally applied tool in earthquake proof design, and in many other problems[1]. After ten years Housner has invented the concept of response spectrum for elastic design of structures in 1941 [2]. Blume and Newmark has developed this concept and create a solid backbone to use it as a functional tool for routine design process of structures [3]. Velestos conducts studies on implementing response spectrum method in inelastic design of structures under seismic loads [4]. By inspiration from John Blume's reserve energy technique , Freeman invented the capacity spectrum method [5] for assessment of nonlinear behavior of structures. All of aforementioned methods and studies are applicable for structural systems subjected to seismic loads. A few studies have been conducted during past years in order to convert these methods into a practical format that can be used for analysis and design of offshore structures subjected to wave loads. Velestos is pioneer in this field, in 1983 he conducted feasibility study on the utilization of response spectrum concept for designing offshore structures against random waves by applying the respective simplifying assumptions [6] in other words he tries to convert response spectrum method into practical tool for design and analysis of fixed offshore structure. Motivated by Veletsos studies, in 1986 Tung demonstrated that one can obtain response spectrum of the offshore structures for an equal single degree of freedom model, by applying the deterministic wave theory and linearization of the drag term [7]. In later years other researchers studied the concept of response spectrum ,such as Hu and et.al [8], who has developed a design response spectra against random ocean waves.

2. Concept of Impulse Response Spectrum

Apart from solving differential equations of motion there are other ways to find response of dynamic system used by vibration theory. One of them is finding the spectral response for excitation or dynamic force exerted in mass of the structure. Each response spectrum represents the nature and characteristics of dynamic load excitation. Response spectrum of impulse load has been studied and used thoroughly since it is base for time domain analysis on the other hand, in the frequency domain it can be used to specify impact load to simulate different scenarios of loading in order to find the maximum response. To illustrate this type of analysis it is essential to acknowledge substantive difference between the excitation in mass of the structure or base of structure. According to Newton's second law, acceleration applied to the base of the structure will convert into inertia force in mass of the structure which is time dependent. One of the fundamental ways of studying and finding the response to the time dependent forces is using short pulses of impact force. In the following figure, difference between excitation in mass and excitation in base of the structure is illustrated.

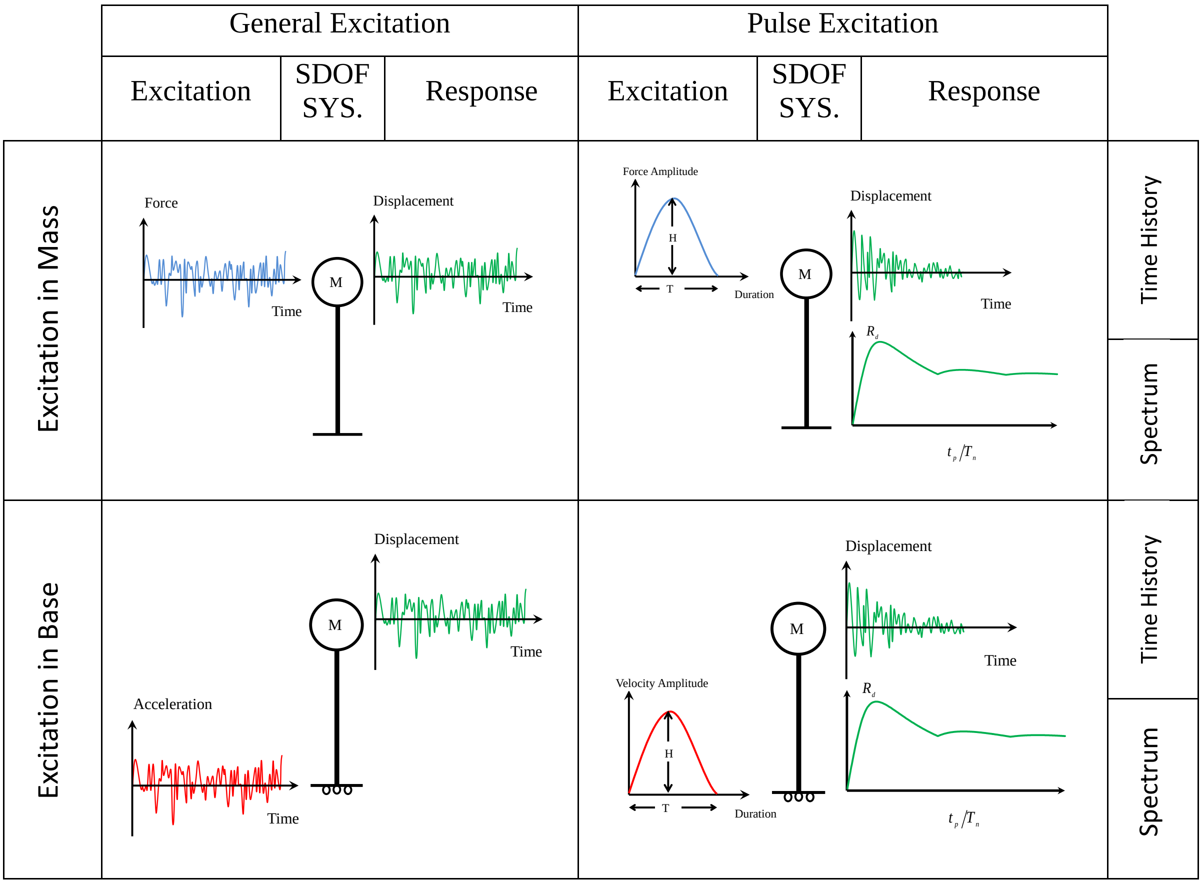


Figure 1: Difference between general excitation and pulse excitation

In many loading situations the excitation is neither harmonic nor periodic thus we must study the dynamic response of system which its excitations is varied with time. The response of structure to each pulse regardless of its form is consist of two different phase. Phase one is force vibration phase, the response of structure in this phase is obtained by Duhamel integral, phase two is free vibration phase and the response of structure in this phase is obtained by solving the differential equation of motion. In the following table the common type of impulse load used in dynamic analysis is represented.

3. Methodology

Two main approaches can be considered for using this concept in assessment of offshore structures. In first approach all mechanical properties of the structure are constant while the period of the pulse is varied. In sinusoidal pulse spectrum while the ratio of t_p/T_n is less than one, increasing the t_p results in the increased response. In section where ratio of t_p/T_n is greater than one, increasing the t_p results in short decreasing response and after that response becomes somehow constant.

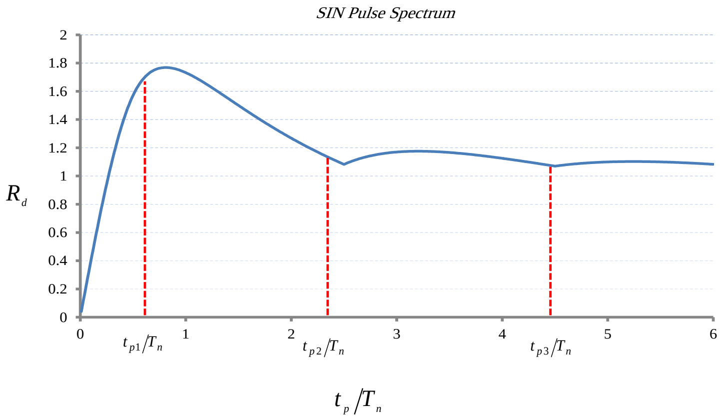


Figure 3: Pulse response spectrum with variable pulse period and constant natural vibration period of the structure

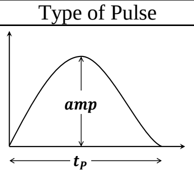
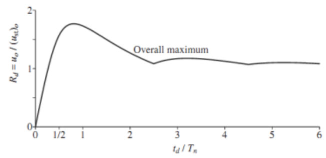
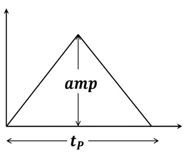
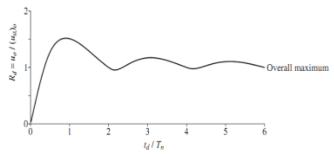
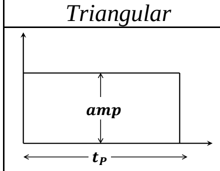
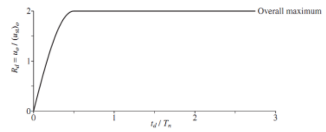
Type of Pulse	Response to pulse formula	Pulse spectrum	Max condition
 *Sinosoidal*	$R_d = \frac{u(t)}{(u_{st})_0} =$ $\frac{1}{1-(T_n/2t_p)^2}\left[\sin(\frac{\pi t}{t_p}) - \frac{T_n}{2t_p}\sin(\frac{2\pi t}{T_n})\right] \quad t \le t_p$ $\frac{(T_n/t_p)\cos(\pi t_p/T_n)}{(T_n/2t_p)^2-1}\sin\left[2\pi(\frac{t}{T_n}-\frac{t_p}{2T_n})\right] \quad t \ge t_p$		$R_{d\max} \simeq 1.76$ $\frac{t_p}{T_n} \simeq 0.81$
 *Triangular*	$R_d = \frac{u(t)}{(u_{st})_0} =$ $2(\frac{t}{t_p} - \frac{T_n}{2\pi t_p}\sin(\frac{2\pi t}{T_n})) \quad 0 \le t \le \frac{t_p}{2}$ $2\left\{1-\frac{t}{t_p}+\frac{T_n}{2\pi t_p}\left[2\sin\frac{2\pi}{T_n}(t-\frac{t_p}{2}) - \sin\frac{2\pi t}{T_n}\right]\right\} \quad \frac{t_p}{2} \le t \le t_p$ $2\left\{\frac{T_n}{2\pi t_p}\left[2\sin\frac{2\pi}{T_n}(t-\frac{t_p}{2}) - \sin\frac{2\pi}{T_n}(t-t_p) - \sin\frac{2\pi t}{T_n}\right]\right\} \quad t \ge t_p$		$R_{d\max} = 1.5$ $\frac{t_p}{T_n} = 1$
 *Rectangular*	$R_d = \frac{u(t)}{(u_{st})_0} =$ $1-\cos\frac{2\pi t}{T_n} \quad t \le t_p$ $(2\sin\frac{\pi t_p}{T_n})\sin\left[2\pi(\frac{t}{T_n}-\frac{t_p}{2T_n})\right] \quad t > t_p$		$R_{d\max} = 2$ $\frac{t_p}{T_n} \ge 0.5$

Figure 2: Different type of Pulse and their close form relation

In second approach the pulse period is constant while the natural vibration period of the structure is variable. In this situation while the ratio of t_p/T_n is less than one, increasing the T_n results in decreased response. In section where ratio of t_p/T_n is greater than one, increasing the T_n results in constant response until the ratio of t_p/T_n becomes 2 after that the response will increase until the ratio becomes 1.

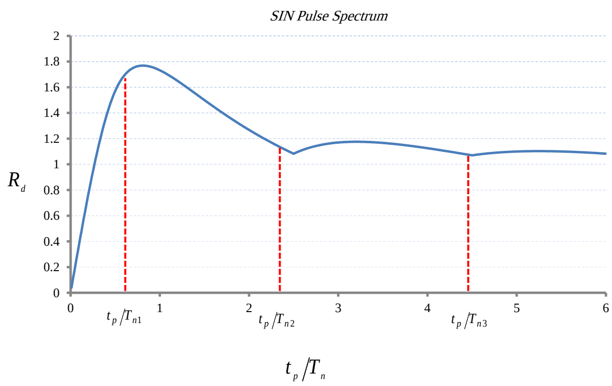


Figure 4: Pulse response spectrum with Constant pulse period and variable natural vibration period of the structure

4. Modeling scope

In this research numerous response spectrum was obtained in order to clarify the idea. In a routine response spectrum used in earthquake engineering the response of single degree of freedom against the natural vibration period or frequency of structure is plotted. In formulation of seismic load the response of structure is independent of structural geometry while in in wave load, according to Morrison equation the response of structure depends on structural geometry therefore response spectrums represented in this study are based on variable structural geometry by means of diameter. Different geometry is defined by different diameter of structural member; the thickness of structural member is defined as 1/40 of diameter. For the sake of simplicity diameters of 70, 80, and 90,100,110,120 is considered for the models studied. The height of structural member is 1 meter. The model is connected to the ground by 3 spring element in which their stiffness is tuned so the natural vibration period of 1.5, 1.6, 1.7, 1.8, 1.9 and 2 second is obtained for the structural member. The selected range of natural vibration period is chosen based on typical natural vibration period of offshore structures located in Persian Gulf. Ocean Waves considered in this study are extracted from typical sea states reported for the Persian Gulf region with the specific height and return period. With a variety of models mentioned, a total of 288 analyses on simple systems have been performed.

Tabale1: Sea states information for Persian gulf

Sea state	specific height (Meter)	Wave Period (Second)	Return Period (Year)
1	1.67	3.8	1
2	2.82	4.94	2
3	3.62	5.6	5
4	4.15	6.0	10
5	4.66	6.35	20
6	5.33	6.78	50
7	5.83	7.1	100
8	6.32	7.4	200

5. Equalization of wave force to a pulse force

In order to obtain a Response spectrum of fixed offshore structures caused by extreme waves and based on concept of impulse response spectrum first we must equalize the ocean wave force to simple Sinosoidal pulse with a specific period and amplitude in other word the period and amplitude of the pulse is computed based on hydrodynamic specification of waves. In order to achieve this goal first we must consider a proper simplified structural model.

Assume a simple one degree of freedom structure with a lumped mass subjected to simple airy wave[9]. As shown in figure 1, platform displacement in height z at time t is denoted by. Ocean wave is modeled by airy wave theory in the water depth "d", wave amplitude "a", wave number "k" and frequency "ω". Accordingly, the water particle velocity and acceleration are calculated based on equation (1) and (2), respectively:

$$u(z,t)=\frac{agk}{\omega}\frac{\cosh kz}{\cosh kd}\cos\omega t \qquad (1)$$

$$\dot{u}(z,t)=-agk\frac{\cosh kz}{\cosh kd}\sin\omega t \qquad (2)$$

The wave load is computed through Morrison's equation. The equation consists of terms drag f_D and inertia f_I :

$$f_D(z,t)=(\frac{1}{2}C_d D(u(z,t)-\dot{z})\,|u(z,t)-\dot{z}\,|) \qquad (3)$$

$$f_I(z,t)=\rho C_m(\frac{\pi D^2}{4})(\dot{u}(z,t)-\ddot{z}) \qquad (4)$$

$$(M_{deck}+M_{st})\ddot{z}+K_{st}z=f_I(z,t)+f_D(z,t) \qquad (5)$$

$$\left(M_{deck}+M_{st}+\rho\left(C_m-1\right)\pi\frac{D^2}{4}\right)\ddot{z}+kz= \frac{1}{2}C_d D\left(u(z,t)-\dot{z}\right)\left|u(z,t)-\dot{z}\right|+\rho C_m\frac{\pi D^2}{4}\dot{u}(z,t) \qquad (6)$$

In all analyzes, added mass generated in the system due to dynamic displacement of the vertical pipe is calculated and considered its impact on the response. In this way, the effect of the acceleration of the particles relative to the tube is considered in the calculation. On the other hand, the relative velocity of the system without damping is neglected since it has little effect on the response.

6. implementation of method

A non-linear differential equation (Equation 8) for each of the 288 different model developed in Matlab, have been used to plot the results. For this purpose, according to Morrison's equation dependence on the height of the structure the total height of the tube is divided into 5 segments and the total force of the wave is obtained by numerical integration of Morrison equation (Force per unit length).

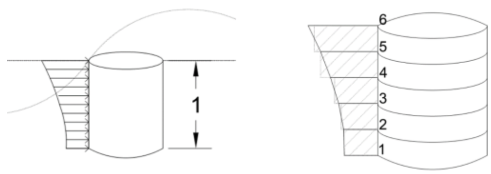


Figure 5: Wave Force is calculated by numerical integration of the Morison's equation

Once again, the same system has been analyzed against the half sine pulse with specific amplitude and period. In all cases, half sin pulse with period equal to half of the airy wave period is considered. The amplitude of the half sin pulse is obtained so the maximum response of the structure against the airy wave load becomes equal to maximum response of the structure against the half sin pulse. With this approach we have replaced a wave force with a short pulse from the pulse spectrum with known answer.

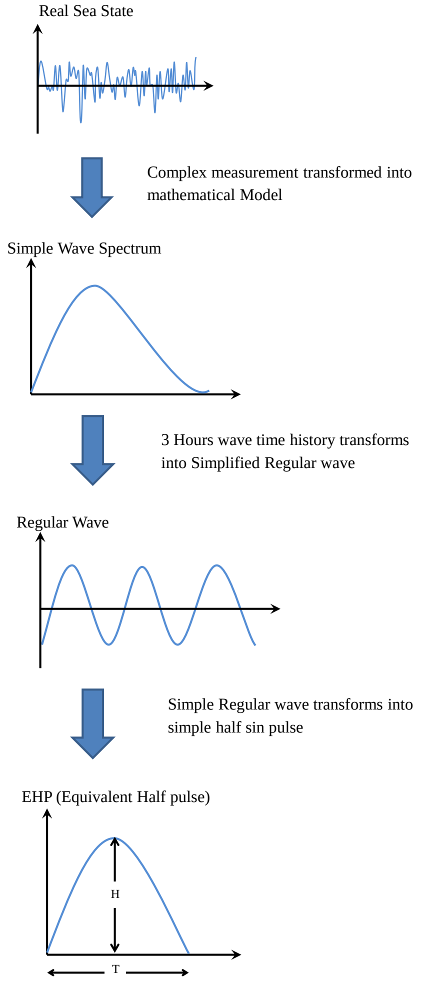


Figure 6: Conversion of Complex environmental loading into simple half sin pulse

7. results and discussion

Due to the regular wave theory, the response time history is consist of two dominant periods, one concerning the nature of the structure and the other period due to the variety of sea wave.

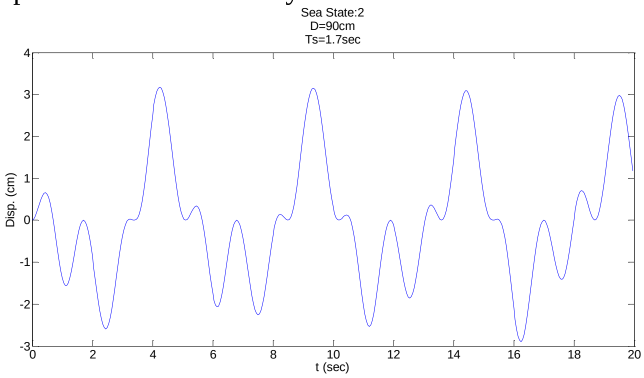


Figure 7: Response time history of the natural period of 1.7 seconds, the pipe diameter of 70 cm and a wave of the sea state 2

In process of structural assessment, the maximum response is always in favor[10]. In spectral point of view the following diagrams can be extracted from maximum response obtained from dynamic analysis. For example, three-dimensional diagram of displacement response spectrum versus pipe diameter and wave period, for structures with natural vibration period of 1, 9 is shown in figure 8. We can conclude that the maximum response in intense sea states is more sensitive to structural member's geometry. This subject is Justifiable considering the importance of inertia term in determination of force exerted on structural member with large diameter. Regarding the equation 4 the inertia load is proportional to square of diameter and compared to drag force that has proportionality to diameter, it has more dependency on geometry.

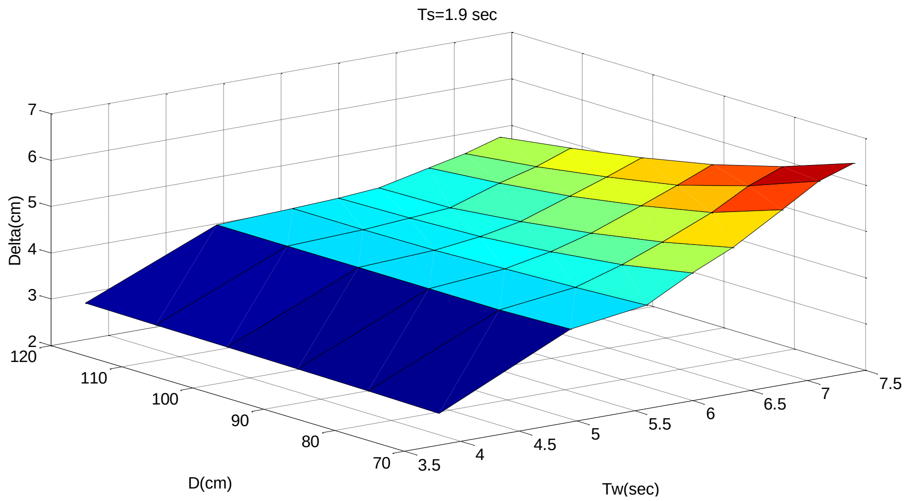


Figure 8: Range three-dimensional structures of the normal period of 9.1 seconds.

By more comprehensive approach, the response can be depicted as ratio of natural vibration period to period of wave versus ratio of diameter of structural member to wave height. In figure 9 this approaches has been shown for sea state 3. More response fluctuations can be seen in this diagram for structures with smaller diameters while the changing rate of response for structures with larger diameters is somehow linear.

Variety of spectrum diagram can be defined for each group of data categorized based on diameter, natural vibration period, wave period, wave height or any meaningful ratio of them serve as input against response like displacement, spring force, energy and RAO…etc.

In this section a spectral approached has been used in order to explain the amplitude of half sin pulse. At first diagram of F0 versus natural vibration period of structure in different sea states has been showed. F0 is half sin pulse amplitude that will equalize the maximum response obtained from wave analysis to maximum response obtained from half sin pulse.

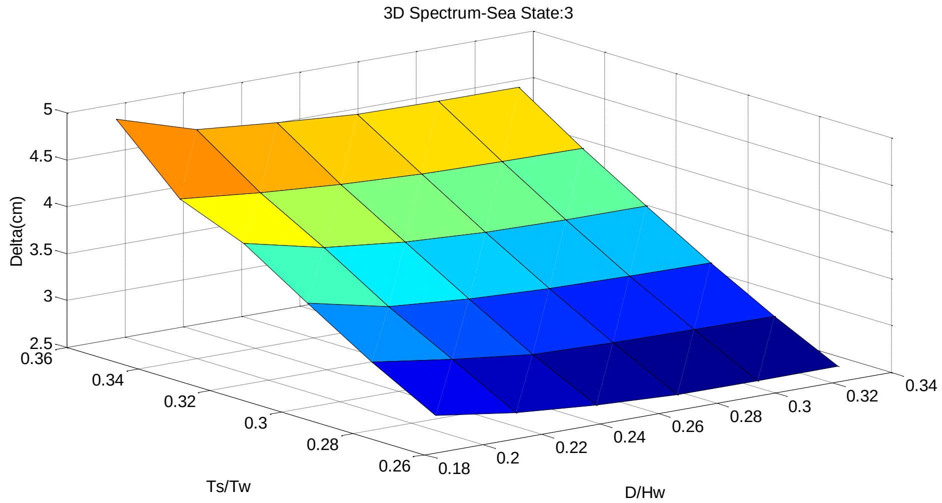


Figure 9: Three-dimensional spectrum of the waves generated by the Sea 3

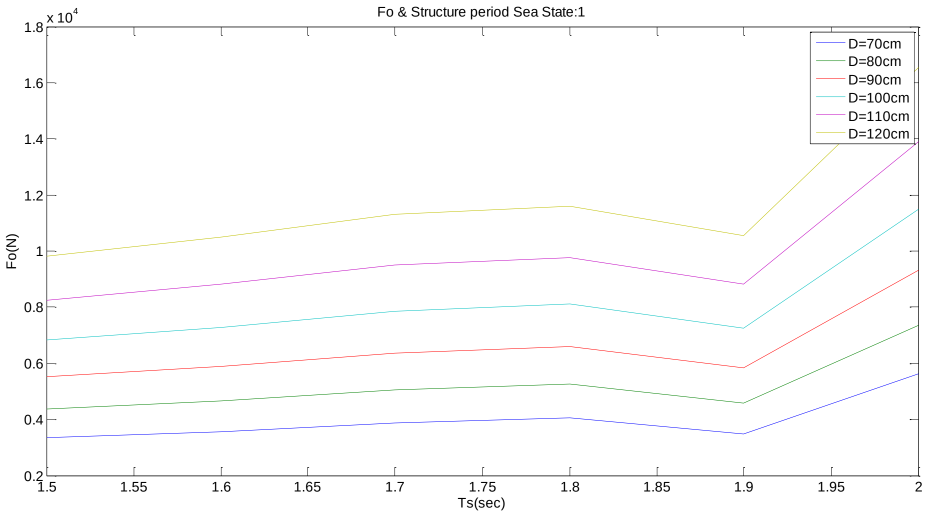


Figure 10: Half the pulse amplitude of the waves generated by the Sea state 1

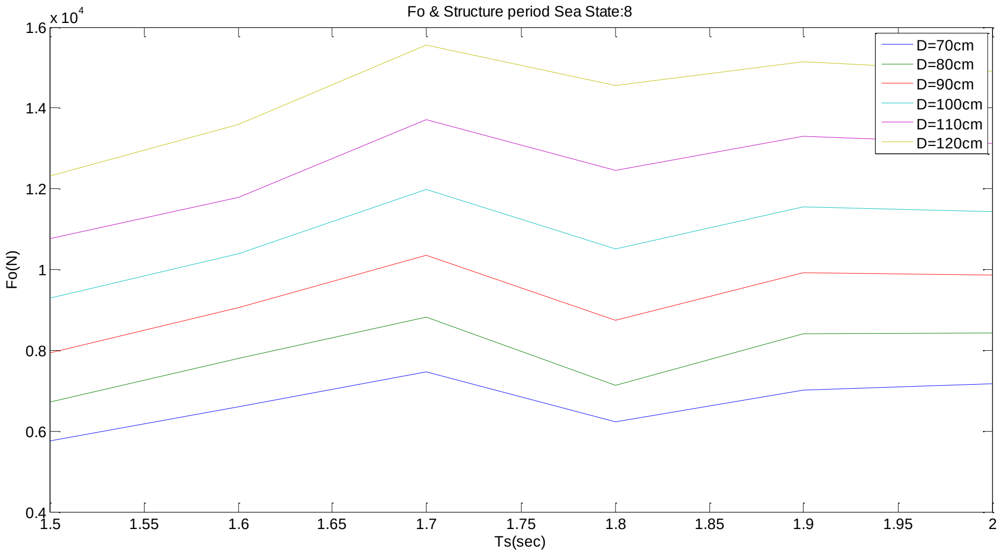


Figure 11: Half the pulse amplitude of the waves generated by the Sea state 8

By looking at above diagram, the data can be assessed from two points of view. The first one is "what is the effect of variable diameter on the EHP for the structures with same natural vibration period in the same sea states?" From this diagram it is clear that in the fixed natural vibration period (a line parallel to Y-axis) increasing the diameter results in larger EHP. Since the increasing of diameter means higher wave force (increasing the drag and inertia load), this trend is prompted. The second one is" what the effect of variable natural vibration period on the structures with same diameter in the same sea state is?" Answering this question requires more accurate reviewing. In general the increase of natural vibration period results in larger EHP but there are some points that are disrupting the trend of change. That's because, with increase of period and decrease of stiffness, the wave force is increased with nonlinear pattern and this phenomenon results in larger displacement in the model and for this reason larger EHP is needed for equalization. Centralization of extreme values related to all structures with different diameters at certain periods could indicate the need for sensitivity in the equalization of a non-linear analysis and a linear analysis in specific points.

8. Summary and conclusions

Analyses offshore structures under environmental loads are very time consuming and therefor simplified methods for estimation of acceptable response of these structures can be very useful in initial design. Concept of response spectrum is one of those simplified methods that is used for seismic analysis of structures specially fixed offshore structures. This method is fully applicable for structures since it is structural independent and only depends on dynamic specification of structure but this concept cannot be used for wave loads since the amplitude of the load is depend on structural geometry and therefor spectrum obtained for offshore structures must be adjusted for variety of geometry type, loading level and etc. In this paper a new method for reducing the cost of analysis for structural design and assessment of offshore structures under wave loads is proposed and it was shown that by equalizing the wave force to half sin pulse and using pulse response spectrum we can obtain scope of response spectrums adjusted for structure that we want to design. In order to obtain these response spectrums first we simplified the structural system then we equalize the regular wave force to half sin pulse by finding amplitude that equalizes the maximum response of half sin pulse to maximum response obtained from regular wave analysis.

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