A Hydrodynamic Model of Tidal Current in the Strait of Hormuz

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ABSTRACT

This research presents a model for prediction of tidal currents in shallow sea areas. This hydrodynamic model is based on the solution of shallow water equations. Also contains effects of earth rotation, topographic changes, and influences of bottom friction. One of the results of solving these equations is the dispersion relation of tidal wave in the study area by which the wave number of each tidal constituent is obtainable. Finding velocity and direction of tidal currents is among other results of solving these equations. Thus, for low-amplitude waves in shallow water, the direction and velocity of tidal currents can be calculated hourly and on different days throughout the year. In order to facilitate calculations, a computer program was written in C++ programming software by which the tidal currents in the Strait of Hormuz have been computed at different times and the results have been diagrammed accordingly. This research indicates that the average velocity of tidal currents at the sea surface of the Strait of Hormuz during Spring tide is 0.35 m/s and ranges between 0.02 m/s - 1.7 m/s. The maximum tidal current velocities occur in shallow coastal waters, whereas in other areas these values are lower. As a result, the range of tidal currents in the Strait of Hormuz is more than that of the Persian Gulf. In addition, results showed that the tidal current direction upon entrance and exit in the Strait indicates the anticlockwise circulation of currents in the Persian Gulf.

1. Introduction

The Persian Gulf in South of Iran is connected to the Oman Sea and Indian Ocean through the Strait of Hormuz with a width of 60 km [1]. Today, this strait is one of the world’s most sensitive and vital waterways. This strait is crescent-shaped 187 km long with a minimum width of around 55km from Ras Qabr al Hindjin in the North-East of Oman Peninsula to the mouth of the Gaz River in Iran and its medium width from Bandar Abbās in Iran to Ra’s-e-Shurit in the North of Oman is about 187km. Its depth, due to the sharp slope of the Strait floor, varies from north to south and is about 36 m near Larak Island while in the southern coasts near Musandam Island, Oman, ranges between 180 to 200 m. The deepest point in the Strait is a pit with 223m depth, which is located 45 km North of Musandam Island.

The Strait of Hormuz as an international strait which is located between Iran and Oman is economically and strategically important. The narrowness of the Strait and existence of several islands and littoral elevations in the vicinity of marine routes make meeting the security requirements of the oil tankers and trading vessels passing through the Strait of Hormuz an internationally important issue [2]. Mehrfar et al using COHERENS model as a three dimensional hydrodynamic model to address the coastal currents in the western Persian Gulf. The obtained results suggested that Iranian northwestward coastal currents developed from January to April and experienced their maximum intensity from June to August when the rate of the surface inflow current increases through the Strait of Hormuz and the stability of the seasonal thermocline becomes gradually stronger [3].

Pous et al. (2015) concluded that thermohaline structure and circulation also vary on intraseasonal timescale, induced by the high-frequency tidal and atmospheric forcings [4]. Mehrfar et al as result of the simulation by three-dimensional hydrodynamic model (COHERENS) showed a coastal jet stream with a speed rate of 30 cm/s from May to October [5].
Le Provost suggested a model for tides at Kuwait coastlines and tidal charts and velocity diagrams of the tidal currents in the Persian Gulf were generated, accordingly. The velocity of tidal currents is mostly 20 cm/s and won't exceed more than 0.4 m/s. It is intensively influenced by the coast topography and seabed type. According to his model, all the predicted tidal currents in the medium water depth have 10 cm/s error and for the maximum tidal range, the current velocity ranges between 1-2 m/s [6].

Lardner et al calculated the current driven by average monthly wind drift and density gradient for the surface and bottom of the Persian Gulf. As held by this research, these currents in the Persian Gulf rotate anticlockwise and in the South-Eastern coasts have the highest values. In Iranian coasts these currents at the sea surface and at the near bottom are about 0.05 m/s and 0.04 m/s, respectively. Also, in Iranian coasts of the Persian Gulf the maximum current generated by average monthly wind drift in June at the sea surface is almost 0.085 m/s and bottom are 0.1 m/s [7].

Floating vessel registers have recorded that there are currents along the Iranian coasts towards the northwest with velocities higher than 0.1 m/s. These currents tend to flow in 51.5 E ° direction (near shore) and change their direction toward the south west of the Gulf away from Iranian coasts [8, 9]. According to Torabi Azad a numerical model for mesoscale motion in the Persian Gulf presented and concluded that the average annual surface currents in the Persian Gulf are between 0.075m/s to 0.1875 m/s [10]. Based on a nonlinear barotropic model for the study of water circulation by wind pressure and tide in the Persian Gulf, was found that the velocity distribution in the Persian Gulf is in a way that water circulations with higher velocities belong to shallow areas and occur along the coastline.

Also, the topographic effects of the Persian Gulf upon nonlinear barotropic currents are highly significant [11]. Teubner et al presented a model that predicts tidal elevations and current velocities which is developed and applied to the Persian Gulf. The model uses finite difference techniques applied to two-dimensional spherical coordinate equations that govern tidal movement in coastal regions. Because of the importance of the Gulf to the shipping and fishing industries, it is necessary to be able to predict tidal elevations and flows at many near shore lines. However, due to the size of the Gulf, it would be impractical to use a very fine finite difference grid over the whole Gulf. Thus, a technique is developed for nesting a fine grid within a coarse grid, so that important areas can be modeled more accurately [12]. The purpose of this study is to provide a method for computing direction and magnitude of a tidal current and describing the effect of depth upon these quantities. The effects of the rotation of the earth and bottom topography are also included in this model.

Results of this research can be used to study effect of tidal currents upon sedimentation and erosion of coastal areas and tidal rivers of the region studied. Furthermore, in designing coastal structures, taking the direction and velocity of tidal currents into consideration is important.

2. Governing Equations

The current can be described by governing equation of force per unit mass (momentum equation) and the continuity equation [13]. These equations are as follows:

\[
\frac{d\vec{U}}{dt} = -\frac{1}{\rho} \nabla P - 2\vec{\Omega} \times \vec{U} + \vec{g} + \Phi
\]

\[
\nabla \cdot \vec{U} + \frac{1}{\rho} \frac{\partial \rho}{\partial t} = 0
\]

Where \( \vec{U} = u\vec{i} + v\vec{j} + w\vec{k} \) is vector velocity, \( \rho \) is the water density, \( \vec{\Omega} \) is the angular velocity vector of earth, \( \vec{g} \) is acceleration of gravity, \( \Phi \) is the other effective forces which is bottom friction here, \( \nabla P \) is pressure gradient and \( \frac{d}{dt} \) is the substantial operator.

Parametric condition which is characterizes shallow-water theory \( \frac{H}{L} \ll 1 \) in study area is considered, sea water is incompressible \( (\nabla \cdot \vec{U} = 0) \), the fluid is barotropic \( (\frac{\partial \rho}{\partial x} = \frac{\partial \rho}{\partial y} = 0) \), tidal waves are small amplitude \( \eta_o \ll H \), the bottom friction is assumed as

\[
\vec{F}_b = k'\rho_w \frac{\vec{U}}{H}, \quad \text{where} \quad k' \text{ is the bottom friction coefficient and equals } 75\times10^{-4} \text{ m/s} [11].
\]

Also, depth variation is taking net in to consideration. In above assumptions, \( H \) is mean water depth, \( L \) is wavelength of tidal constituent, \( \eta \) is the amplitude of tidal constituent, and \( H_i \) is station depth from mean sea level.

According to the above assumptions tidal waves are small amplitude waves moving in shallow water. Further we suppose \( u \) and \( v \) (velocity components of East -West and North - South direction) are small enough that \( \frac{\partial U}{\partial t} \gg \nabla \vec{U} . \nabla \vec{U} \). Also, from the dimensional analysis \( \nabla . \vec{U} = 0 \) it can be concluded that, there is no need to calculate the vertical velocity component because it can be ignored compared with the values of other components.

The primary expansion of the governing equations is as below:
\[
\frac{\partial u}{\partial t} + \left( \frac{u}{\partial x} + \frac{v}{\partial y} + \frac{w}{\partial z} \right) = -f v - \frac{1}{\rho} \frac{\partial p}{\partial x} - k' \frac{u}{H_0} \\
\frac{\partial v}{\partial t} + \left( \frac{u}{\partial x} + \frac{v}{\partial y} + \frac{w}{\partial z} \right) = \frac{1}{\rho} \frac{\partial p}{\partial y} - k' \frac{v}{H_0} \\
\frac{\partial w}{\partial t} + \left( \frac{u}{\partial x} + \frac{v}{\partial y} + \frac{w}{\partial z} \right) = -\frac{1}{\rho} \frac{\partial p}{\partial z} - g
\]

(3)

Where \( f \) is Coriolis parameter, \((u, v, w)\) are the velocity components in the eastward \((x)\), northward \((y)\) and vertical \((z)\) directions respectively. From the \( z \) component of above equations (3) the pressure relation can be obtained in terms of the depth and from that the pressure displacement can be related to the displacement of oscillation dimension of the tidal constituents. Considering the above assumptions and explanations, a partial differential equation system is obtained as follows:

\[
\begin{align*}
\frac{\partial u}{\partial t} - f v = -g \frac{\partial \eta}{\partial x} - k' \frac{u}{H_0} \\
\frac{\partial v}{\partial t} + f u = -g \frac{\partial \eta}{\partial y} - k' \frac{v}{H_0} \\
\frac{\partial w}{\partial t} + \frac{w}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial z} - g
\end{align*}
\]

(4)

The Solutions (4) maybe sought in form of a plane wave, i.e [14].

\[
\eta = \eta_0 \cos (kx + ly - \sigma t)
\]

(5)

Where \( \eta \) is the tide height due to a particular constituent and its phase is \( kx + ly - \sigma t \).

\( \vec{K} = k \hat{i} + l \hat{j} \) is wave propagation vector, \( k \) and \( l \) are wave vectors of each tidal constituent in North - South and East –West directions, respectively. \( K \) is wave number vector; \( \sigma \) is the angular velocity of the tidal constituent.

If (5) is substituted in (4) after some manipulation, yields the dispersion relation for each tidal constituent is as follows:

\[
\sigma^2 = \left( \frac{k'^2}{H_0^2} + f^2 \right) + C_v^2 K^2
\]

(6)

Here \( C_v = \sqrt{g H_0} \) is the phase velocity. The velocities \( u \) and \( v \) can be found in terms of \( \eta \) from the solution of differential equation system (4):

\[
\begin{align*}
\eta &= \frac{2 \eta_0 f}{H_0^2} \frac{f^2 - \sigma^2 + k'^2}{H_0^2} x K \cos(kx + ly - \sigma t) + \\
&+ \frac{2 \eta_0 f}{H_0^2} \frac{f^2 - \sigma^2 + k'^2}{H_0^2} x K \sin(kx + ly - \sigma t)
\end{align*}
\]

(7)

The velocity component is parallel to wave propagation vector \( u_\parallel \) and velocity component is perpendicular to the wave propagation vector \( u_\perp \) are as follows:

\[
\begin{align*}
\eta_j &= \frac{2 \eta_0 f}{H_0^2} \frac{f^2 - \sigma^2 + k'^2}{H_0^2} x K \cos(kx + ly - \sigma t) + \\
&+ \frac{2 \eta_0 f}{H_0^2} \frac{f^2 - \sigma^2 + k'^2}{H_0^2} x K \sin(kx + ly - \sigma t)
\end{align*}
\]

(9)

It is convenient to replace

\[
\eta = \frac{f \eta_0}{2} \cos \left( \frac{2 \pi}{T} t + g' + V_0 \right)
\]

in (5) and applying in (9) and (10) [15]. Where \( f' \) is knot factor, whose value for each day should be taken from tide tables and can be used as a coefficient for the correction of the amplitude, \( V_0 \) is the astronomical argument which is printed for important constituents such as Admiralty Tide Table [16]. \( T \) is period of tidal
constituent, $g'$ is the phase delay which is calculated regarding the tidal observations. The utilized depth values are extracted from the depth maps of the Geography Organization of the Army and the amplitude and Greenwich phase delay of each station are extracted from the co-tidal maps of the Cartography Organization of Iran [17]. In order to made the calculations a computer program was written in C++ and was run in two stages:

A) At the first stage, by the use of the instructions available in Admiralty Tide Tables, a computer program is written to help interpolate the knot factor and astronomical arguments for each constituent in the related region [16, 17]. The output of the present program will be an input file for the second program.

B) At the second stage, by the use of what we described before a program is written whose output will be the velocity and direction of the tidal flow.

3. Results and Discussion

In order to verify the equation result and also output of current velocity which has determined by computer, the output of this program is going to compare and study with the existent data. This study was done in two steps:

a) Prediction of current in a station in the Strait of Hormuz and comparison with the field study measurements.

b) Prediction of current in two stations with different depths and compare the results by tidal theory. Location of study stations are presented in figure 1.

1-3 Prediction of current at a station in Strait of Hormuz in accordance with the field Current measurement

For this step, station 1 were considered in the Strait of Hormuz, then the output mean velocity of prediction program, with the mean velocity of field Current measurement in this station has been compared. It’s necessary to mention those current meters are floated half meter from the water surface (Table 1). Program outputs with the mean velocity which obtained from computer program running are shown in Table 2. The mean velocities in table 1 and 2 have been compared with each other. It is remarkable that there is a significant difference between two mean velocity measures in these stations which caused by current meters floating or currents arise by wind, currents arise by different density and rivers discharge in this area. The difference between two average velocities can be due to the instrumental errors in field measurements or the approximations used in the calculation of tidal currents.

2-3 Prediction of current in stations with variation depths and results comparison by tidal theory

In this step, current variations with depth in stations 1 and 2 near the shore has been studied, the depth was less than 20m. In other hand the output program verify with the tidal theory has been checked (Table 3and 4).
Table 1. Information of current monitoring

<table>
<thead>
<tr>
<th>Time specification of current metering</th>
<th>Longitude (Deg)</th>
<th>Latitude (Deg)</th>
<th>Velocity (cm/s)</th>
</tr>
</thead>
</table>

Table 2. Prediction of tide characteristics in station 1

<table>
<thead>
<tr>
<th>Longitude (Deg) :56.18 E</th>
<th>Latitude (Deg) :26.41 N</th>
<th>Depth (m) :85.73</th>
<th>Velocity (cm/s) :16.58</th>
</tr>
</thead>
<tbody>
<tr>
<td>M2</td>
<td>S2</td>
<td>K1</td>
<td>O1</td>
</tr>
<tr>
<td>Amplitude (m) Phase(deg)</td>
<td>Amplitude (m) Phase(deg)</td>
<td>Amplitude (m) Phase(deg)</td>
<td>Amplitude (m) Phase(deg)</td>
</tr>
<tr>
<td>0.71</td>
<td>313.5</td>
<td>0.25</td>
<td>357</td>
</tr>
</tbody>
</table>

Table 3 Tide characteristics of station 1

<table>
<thead>
<tr>
<th>Tidal constituent</th>
<th>Amplitude (m)</th>
<th>Greenwich delay phase (Deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M₂</td>
<td>0.93</td>
<td>312.56</td>
</tr>
<tr>
<td>S₂</td>
<td>0.3</td>
<td>356.59</td>
</tr>
<tr>
<td>K₁</td>
<td>0.3</td>
<td>69.31</td>
</tr>
<tr>
<td>O₁</td>
<td>0.2</td>
<td>56.94</td>
</tr>
</tbody>
</table>

Table 4. Tide characteristics of station 2

<table>
<thead>
<tr>
<th>Tidal constituent</th>
<th>Amplitude (m)</th>
<th>Greenwich delay phase (Deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M₂</td>
<td>0.91</td>
<td>309.33</td>
</tr>
<tr>
<td>S₂</td>
<td>0.3</td>
<td>342.07</td>
</tr>
<tr>
<td>K₁</td>
<td>0.3</td>
<td>64.75</td>
</tr>
<tr>
<td>O₁</td>
<td>0.2</td>
<td>50.25</td>
</tr>
</tbody>
</table>

Characteristics which mentioned in Table 3 are the inputs of program. Current for 25th of January 2016 at surface of water is predicted. So, the magnitude and current direction can be individually studied. Figures 2 and 4 shows time variation of current magnitude respectively at two stations 1 and 2 that predicted by the model. It is expected that as tides are due to a harmonic force, changes in water height as well as changes in current velocity are harmonic. Figures 3 and 5 shows time variation of current direction respectively at two stations 1 and 2 that predicted by the model.
Figure 2. Time variations of current magnitude at sea surface (station 1)

Figure 3. Time variation of Current direction at sea surface (station 1)

Figure 4. Time variations of current magnitude at sea surface (station 2)
It is considering that current vector is parallel with the strait axis or west-east axis in maximum hours of a day and also current direction is changing when spring tide shift to neap tide. If this program runs for deeper stations, it is found that in these stations, has a high difference between tidal theory and model results. Thus, by calibration in selected stations and change of friction coefficient, it can be possible model results approach to result of theory.

Results of this model have been shown a good precision of tidal currents at upper layer and sea surface. Subsequently, by this program the tidal current for the surface of the Strait of Hormuz will be predicted. For this purpose, an area measuring 200 km in length and 100 km in width was chosen so that it encompassed the Strait of Hormuz. Then, at every five kilometers a station was chosen and the characters of each station including the station’s longitude and latitude, depth and tidal characteristics were saved in a file which later served as the input of the computer program. The output of the compiled program was made ready by Excel software, so to use of Surfer software we can draw vector field of tidal current. Tidal current vectors at water surface in several times on 27th June 2017 are shown in figures (6) to (9).

One may notice that 27th June 2017 is beginning time of lunar moon which maximum tide happens, so it is expected that the maxima tidal currents exist in this time. Also, the program was run in each 6 hours according to semidiurnal period of tide.
The predicted of tidal currents indicates that when the flow enters the Persian Gulf through the Strait of Hormuz, it deviates towards North-West and during leaving the Gulf it is flowing toward North-West. This result is compatible to Raees al-Sadat and which are consistent with the results of Sabbagh Yazdi and Mashayekh Poul [1, 18 and 19].

Tidal currents upon leaving the Strait of Hormuz and when reach to shallow Eastern regions and return coastal currents, has flow almost along the coastline. This result is also compatible with research done by Khaleghi Zavareh in 1994 [11].

Both from the predicted values and the figure of tidal currents, it is deduced that stronger currents are in shallow areas, a fact which had verified by the relevant previous studies. Mean velocity of tidal currents at the surface of Strait of Hormuz and during the first 12 hours of a month is 0.35 m/s and in average its velocity varies between 0.02 m/s to 1.7 m/s. The maximum of these current velocities occur in shallow coastal areas. Based on researches done in the Persian Gulf; currents vary from 0.0017 m/s to 0.15 m/s [10, 20]. Therefore, the range of velocity of tidal currents in the Strait of Hormuz is higher than that of the Persian Gulf. Furthermore, it is observed that at the center of the Strait of Hormuz tidal currents are parallel with the Strait’s axis and velocity vectors are also parallel with each other. The analysis of current data shows that semi diurnal tide is the main constituent. Hence this component was removed to consider the residual components due to wind and buoyancy typical deep tidal current is about 30-40 cm/s while the surface tidal current can reach 112 cm/s and the residual current range from 4.5-7.6 cm/s. In research Stations of Strait of Hormuz, residual current magnitude in winter 5 cm/s is smaller than the spring rate 8 cm/s. The effects of tide on currents in the Strait of Hormuz at consider stations have been shown. By moving from the center towards the north coasts of Strait of Hormuz increasing the speed of tidal currents. Residual currents are influenced by wind and density differences and moves from the north of Strait of Hormuz into the Persian Gulf. This current movement speed is lower in winter than in spring. Lardner and et al the monthly mean wind driven current and density gradient current was investigated for Persian Gulf and speed of currents close to the coast of Iran at surface 0.05 m/s to about 0.04 m/s in bottom was estimated [7].

4. Conclusions

A model has been made to predict the tidal currents in the Strait of Hormuz. This model uses the shallow water equations and assumes that tidal waves are small amplitude waves. The solutions of governing equations sought in form of a plane wave, and this leads to dispersion relation for each tidal constituent. The model is run for magnitude and direction of tidal currents in Strait of Hormuz in day that maximum amplitude of tide happens in 2017. Model results show that mean values of tidal currents at sea surface varies between 0.02 m/s to 1.7 m/s. Current vectors being parallel to the flow axis in the center of Strait of Hormuz indicate that tidal currents in the Strait have similar characteristics with currents in narrow channels. It is found that of this study, maximum tidal currents are related to coastal areas and the tidal circulation in the Strait of Hormuz is anticlockwise.

5. References

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